DAMAGE DETECTION USING SUBSTRUCTURE IDENTIFICATION

by

Charles Edward DeVore

A Dissertation Presented to the FACULTY OF THE USC GRADUATE SCHOOL UNIVERSITY OF SOUTHERN CALIFORNIA In Partial Fulfillment of the Requirements for the Degree DOCTOR OF PHILOSOPHY (CIVIL ENGINEERING)

May 2013

Copyright 2013

Charles Edward DeVore

UMI Number: 3563834

All rights reserved

INFORMATION TO ALL USERS The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if material had to be removed, a note will indicate the deletion.



UMI 3563834

Published by ProQuest LLC (2013). Copyright in the Dissertation held by the Author.

Microform Edition © ProQuest LLC. All rights reserved. This work is protected against unauthorized copying under Title 17, United States Code



ProQuest LLC. 789 East Eisenhower Parkway P.O. Box 1346 Ann Arbor, MI 48106 - 1346

Acknowledgements

The author acknowledges the partial financial support of the National Science Foundation (NSF) through various grants: CMMI 08-26634, CMMI 11-00528, CMMI 11-33023, EAPSI 10-15496, and GK12 10-45595. The author also acknowledges the University of Southern California Provost Fellowship and the ARCS Foundation for their generous support.

I would like to thank my advisor, Professor Erik A. Johnson, for his support and guidance throughout my post-graduate education. He has been a tireless advocate and provided many opportunities for me.

I would like to thank my committee members, Professors Sami Masri, Roger Ghanem, and Jerry Mendel, not only for their service, but for inspiring me to be a better researcher. Each one of your classes challenged me and had a profound effect on my intellectual development.

I would like to thank Professor Richard Christenson of the University of Connecticut for allowing me access to his research laboratory. The successful experiment reported herein would never have been possible without your assistance. Thank you to your students Dr. Zhaoshuo Jiang and Mr. Gannon Stromquist-LeVoir for their contributions and expertise.

I would like to thank Professor Yozo Fujino and the researchers at the University of Tokyo for hosting me. The summer in the Bridge and Structures Lab showed me a whole new world of research. I would like to especially thank Mr. Hisashi Yamasaki for his extreme hospitality and friendship.

I would like to thank all of my friends that have graduated before me, Professors Tat S. Fu, Dongyu Zhang, Fabian Rojas, ..., for paving the way and showing me that it can be done. I would like to especially thank Dongyu for his patient explanations and diligent research that formed the basis for so much of the work contained herein.

I would like to thank all of my friends that are struggling with me, (soon-to-be Drs.) Miguel Hernandez-Garcia, Mahmoud Kamalzare, Elham Abiri, Wael Elhaddad, Leonardo Chavez, ... Thank you for making this experience so memorable.

Finally, I would like to thank my family and friends. I would have never made it through this journey without your encouragement. Thank you for your love, support, and all the BBQs.

Table of Contents

Acknow	vledgements	ii												
List of 7	List of Tables List of Figures													
List of l														
Abstrac	t	xi												
Chapte	r 1 Introduction	1												
Chapte	r 2 Literature Review	5												
2.1	Motivation	5												
2.2	Structural Health Monitoring	7												
2.3	Substructure Identification	11												
2.4	Limitations of Current Substructure Identification	21												
Chapte	r 3 Testbed Structure	26												
3.1	Structure	27												
3.2	Response	30												
3.3	Signal Processing	31												
3.4	Monte Carlo Simulation	33												
I The	oretical Developments	36												
Chapte	r 4 Substructure Identification Estimator	37												
4.1	Estimator Formulation	37												
4.2	Nonlinear Function Estimation	46												
4.3	Decentralized Processing	52												
4.4	Estimator Error Prediction	59												
Chapte	r 5 Nonlinear Regression	61												
5.1	Estimation	62												
5.2	Confidence Regions	66												
5.3	Statistical Curvature	73												
5.4	Error Analysis	80												

5.5	Best Practice	87
II Nu	merical Simulations	89
Chanta	r 6 Statistical Parformanca	00
Chapte.	Pice Ennor	90 01
0.1 6.2	Dias Error	91
6.2	Damage Detection	93
6.3		93 97
0.4		71
Chapte	r 7 Controlled Substructure Identification	102
7.1	Mathematical Preliminaries	103
7.2	Uncontrolled System	106
7.3	Controlled System	107
7.4	Observed System	121
7.5	Implementation	126
III Ex	perimental Results	130
Chapte	r 8 Bench Scale Structure	131
8.1	Experimental Apparatus	131
8.2	Control Design	135
8.3	Experimental Procedure	136
8.4	Results	139
<u> </u>		1 10
Chapte	r 9 University of Connecticut Structure	142
9.1		143
9.2	Error Analysis	148
9.3	Experimental Procedure	149
9.4	Results	152
9.5	Preliminary Study of Active Control	159
9.6	Findings	163
Chapte	r 10 Conclusions	165
- 1		
Chapte	r 11 Future Work	167
11.1	University of Connecticut Active Control Experiment	168
11.2	Semi-Active Controlled Substructure Identification	169
11.3	Damage Detection in a Welded Steel Moment Frame	170
11.4	Continuous Beam Estimator for Bridge Structures	171
Referer	nces	173

List of Tables

4.1	Data Transmission	59
5.1	Estimator Performance	67
5.2	Statistical Coverage	72
5.3	Statistical Curvature	79
5.4	Substructure Identification Error Predictions	85
6.1	Bias Results	91
6.2	Variance Results	94
6.3	Damage Scenarios	96
6.4	Statistical Coverage of Damage Detection	97
7.1	Controller Specification	115
7.2	Controller Results	115
7.3	Controller Identification Statistics	120
7.4	Observer Results	124
7.5	Observer Identification Statistics	124
8.1	Bench Scale Structure Nominal Story Parameters	133
8.2	Identified Stiffness Statistics	140
9.1	Nominal Story Parameters	145

9.2	Accelerometer Signal to Noise Ratios	148
9.3	Damage Scenarios	151
9.4	Global Natural Frequencies	157
9.5	Identified Substructure Stiffness	158
9.6	UConn Controller Specification	159
9.7	UConn Controller Results	161
9.8	UConn Controller Identification Statistics	161
9.9	UConn Observer Results	162
9.10	UConn Observer Identification Statistics	163

List of Figures

3.1	Testbed Structure	28
3.2	Testbed Mode Shapes	29
3.3	Testbed Substructure	30
3.4	Identified Parameter Distribution	34
4.1	$H_{\rm EST}$ vs $H_{\ddot{x}_i, \ddot{u}_g}$	45
4.2	Covariance of H_{EST}	51
4.3	Network Topology	56
4.4	Error Prediction Comparison	60
5.1	Least Square Estimator Surface	63
5.2	Maximum Likelihood Estimator Surface	65
5.3	Jacobian of $H_{\rm EST}$	83
5.4	Substructure Identification Error Predictions	86
6.1	Bias Results	92
6.2	Bias Resdiual	93
6.3	Variance Results	95
6.4	Damage Detection Results	98
6.5	Variance Residual	100

7.1	Regulator Block Diagram	103
7.2	Uncontrolled Block Diagram	105
7.3	Controlled Block Diagram	107
7.4	Pole Feasibility	111
7.5	Interstory Acceleration Response for Various Controllers	117
7.6	Interstory Acceleration Response for Various Controllers	118
7.7	Observer Block Diagram	121
7.8	Observers	125
7.9	Class Diagram	129
8.1	Bench Scale Structure	132
8.2	Bench Scale Performance Index	134
8.3	Bench Scale Control 01	138
8.4	Bench Scale Control 02	139
8.5	Identified Stiffness	140
9.1	UConn Structure	144
9.2	PSD of Shake Table Acceleration	146
9.3	UConn Accelerometer Installation	147
9.4	UConn Error Prediction	149
9.5	Damage Detection Results	152
9.6	Substructure Identification for \mathcal{D}_0	153
9.7	Substructure Identification for $\mathcal{D}_1 \dots \dots \dots \dots \dots \dots \dots \dots$	154
9.8	Substructure Identification for \mathcal{D}_2	155
9.9	Substructure Identification for \mathcal{D}_3	156

9.10	UConn Controller	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	160
9.11	UConn Observers																													162

Abstract

As civil infrastructure ages, occupants and users are placed at risk. Due to limited funding, agencies are required to push structures past their original design lifetime. This creates an imperative for the civil engineering community to develop robust and accurate methods for monitoring the health of civil structures and ensuring public safety. This goal is realized by developing methods to detect both long-term degradation and immediate post-event health assessment. New methods are required because current practice, based on subjective time- and labor-intensive visual inspection is unable to adequately meet these needs. This requires novel research to transform the current state-of-the-art of visual inspection into a new paradigm of continuous monitoring.

Substructure identification has emerged as a promising damage detection and longterm monitoring tool for civil structures. Substructure identification starts by applying a reduced order model to a portion of the structure — analogous to a coarse finite element model — and then forms an estimator of the reduced order behavior using response measurements from the global structure. Its benefits are increased sensitivity to common structural damage, decentralized data processing, improved statistical performance, and others. This work develops a generalized framework for formulating substructure estimators. Moreover, it develops two important predictors of estimator performance: model function curvature and an identification error analysis. This allows the analyst to develop an improved estimator and evaluate its performance.

These theoretical developments are applied to several simulations including uncertainty propagation, damage detection, and damage localization. These simulations demonstrate that substructure identification is well-suited for chain structures. Next, a controlled substructure identification procedure is described and the performance is evaluated. An active control law is developed using non-convex constrained optimization.

Experimental verification is provided by two studies. First, a two-story, bench-scale flexible structure is identified. Then, improved identification precision is provided by passive structural control. The second study uses a 12 ft, four-story, steel structure. This structure is identified and damage, caused by releasing a story-level's boundary condition, is detected. Moreover, second-floor identification is not achieved, which is correctly predicted by the identification error analysis developed herein.

Concluding remarks are provided and avenues for future work are detailed. Specifically, an active control experiment using the 12 ft structure is proposed. Semi-active control design is discussed and substructure identification estimators for frame and bridge structures are outlined.

Chapter 1

Introduction

To ensure life-safety and minimize economic costs, damage in civil structures should be detected, located, and quantified at the earliest possible moment. In contrast with many mechanical structures, a civil structure is designed and operated to be life-safe throughout its life-cycle. This means that occupant safety will be held to the highest consideration, earning civil engineers a reputation as conservative practitioners. This results in an aversion to new methodologies and an over-reliance of visual inspection as a tool to detect damage in civil structures.

Visual inspection evolved as a set of inspection procedures to detect common forms of damage. Many federal, state, and local regulations govern damage detection in civil structures. While considerable effort has standardized visual inspection, its application is inherently subjective. Specifically, visual inspection suffers from multiple defects: visual inspection is time- and labor-intensive; it is costly; it is subject to human error; and is limited to detecting visually accessible damage.

There are two classes of inspection methodologies that can be used in combination (or exclusion) with visual inspection: enforced downtime and automated monitoring. Enforced downtime works by specifying a design life of the structure and then decommissioning the structure when that time expires. Likewise, after a large load event (*e.g.* earthquake), enforced downtime would mandate that structures remain un-occupied for a specific time and remove a portion of the remaining design life of the structure. Conversely, an automated monitoring inspection methodology would be condition-based and continuously monitor the health of the structure through a set of features. When one of the damage features indicates that damage is likely to have occurred, the structure will be evacuated. This assessment will occur continuously and can detect damage after a large load event. A more complete discussion of current practice and motivation is provided in Section 2.1.

Many researchers have developed automated damage detection methodologies, which are commonly known as structural health monitoring (SHM). These techniques complement visual inspection and try to overcome its deficiencies. Specifically, SHM overcomes the inherent variability of visual inspection by prescribing a procedure which is datadriven and automated. Several popular techniques are described in Section 2.2.

Among SHM researchers, many have focused their attention on dynamic data. In this approach, the structure being monitored is instrumented with dynamic sensors, the data from which are processed by a variety of algorithms to classify a set of features that can differentiate between a healthy and a damaged structure. Within a subset of these researchers are a group using decentralized algorithms. These methods are of great practical importance because real-world implementations make centralized data processing unlikely, particularly as the availability of inexpensive wireless sensors becomes a reality and dense sensor arrays are utilized. This work will investigate the use of substructure identification as a decentralized method to detect damage in a civil structure. Substructure identification works by considering a portion of the structure and identifying a parametric reduced order model (ROM) that describes the local structural behavior. Specifically, this work will track locally identified stiffness parameters and consider statistically significant deviations to represent damage. To provide context, a comprehensive literature review of substructure identification is provided in Section 2.3.

To evaluate the effectiveness of substructure identification, one fundamental question needs to be answered: To what extent does structural damage cause a detectable change in local stiffness? In answering this question, this work will investigate the statistical properties of the estimator, analyze different structural configurations, and consider different damage scenarios. Predictive error analysis will show which substructures admit better identification performance than others while different model functions can be compared via statistical curvature.

The thesis of this work is that substructure identification does not generate a unique estimator. Rather, many different formulations can be used, each with unique properties. Moreover, the behavior and performance of these different estimators can be predicted through the statistical curvature of the model function and an error analysis of the estimation function. A generalized procedure for formulating and predicting the performance of substructure estimators is described in Chapters 4 and 5 and forms the first major contribution of this work.

Next, this dissertation considers the numerical performance of substructure identification in two ways. Chapter 6 analyzes the uncertainty propagation and damage

3

detection capabilities of the proposed algorithm. Chapter 7 develops a procedure to use an active control device to temporarily change the dynamics of the structure to improve substructure identification performance of a particular substructure.

The final major contribution of this work is the experimental verification of substructure identification in two structures. In Chapter 8, a bench-scale structure is identified using substructure identification. Identification precision is improved by altering the structural configuration which confirms controlled substructure identification. In Chapter 9, damage is detected and located using substructure identification on a 12 ft. four-story steel structure. The findings show that substructure identification is more sensitive to damage than equivalent global methods. Furthermore, observed substructure identification performance was predicted and observed experimentally. This motivates the future work of controlled substructure identification.

This study will conclude with a summary of major findings. A discussion of future work will be presented to outline areas of fruitful research.

Chapter 2

Literature Review

This chapter provides a literature review of topics related to substructure identification. First, the motivation is provided in the context of damage detection of civil infrastructure. Second, a brief discussion of global structural health monitoring techniques is presented. Third is an extensive review of substructure identification methods. Fourth is a discussion of areas where future work is needed.

2.1 Motivation

As civil infrastructure ages, there is growing concern about the safety of the nation's built infrastructure. Signs of deterioration, including the catastrophic collapse of the I-35W Bridge (NTSB, 2008), indicate that civil engineers need to take a new interest in ensuring the structural reliability and safety of civil structures. Quantifying these concerns, the American Society of Civil Engineers (ASCE) published its 2009 infrastructure report card and gave the nation's bridge inventory a "C". Among the reasons given was that one in four bridges are structurally deficient (ASCE, 2009). This report indicates that current construction and spending levels are insufficient to improve this ratio. Therefore, to extend the life cycle of civil infrastructure, it is necessary to find cost effective methods to characterize a structure's health and predict its life expectancy.

In addition to bridge infrastructure, the nation's building inventory is at risk. Like bridges, buildings are susceptible to long-term deterioration caused by fatigue, corrosion, material defects, and so forth. However, a more fundamental concern is excessive loading caused by extreme events such as earthquakes, high wind, and hurricanes. Following an extreme event, it is necessary to evaluate the health of the structure to ensure life safety for occupants. Thus, current practice prescribes that after a strong earthquake a visual inspection will be performed to determine if damage has occurred (ATC, 1989).

Unfortunately, visual inspection is a costly process that suffers several defects. First, visual inspection is both time and labor intensive. ATC-20 (ATC, 1989) indicates that, on average, post-earthquake visual inspection of buildings in an effected area will take 90 days. Likewise, federal regulations specify that bridges should be routinely inspected on a 24-month interval (Bridges, Structures, and Hydraulics, 2011). Many states struggle to meet this requirement, which reflects the time-consuming nature of visual inspection. Second, visual inspection is costly. Specialized training is required to teach structural inspectors and further specialized inspections are often required. Following the Northridge earthquake, buildings with welded steel moment frames needed each welded joint inspected with an average cost of \$1000 per joint (Gates and Morden, 1995). Third, even with uniform code regulations and standardized training, visual inspection is an inherently subjective technique. Many researchers and building officials have criticized visual inspections for this reason. A United States Geological Survey (USGS) report (Bruce and Tubbesing, 1994) finds that visual inspections are a tenuous predictor of structural safety

6

and that many building inspectors are under pressure to conservatively list structures as damaged because of their own uncertainty.

A technique complimentary to visual inspection is design-controlled life cycles and code-regulated downtime following an extreme event. Much of current practice is centered around the idea of the design life of a structure. Within this time-frame, the structure is assumed to be healthy. Also, after an extreme event, it is assumed that the structure is damaged until properly inspected and found undamaged. Yeo and Cornell (2009) developed a dynamic programming method to determine the optimal amount of down-time for a structure following an earthquake. This prescription is found by weighing the uncertainty surrounding life-safety risks, after-shock hazards, and damage transition probabilities. While these methods can be designed to minimize life-safety risks, they are unable to autonomously utilize information about the structure's current condition.

It is clear that further research is needed to address the limitations of current practice in civil damage detection. Civil infrastructure is aging and under risk from extreme events. Current practice relies on visual inspections which suffer from numerous defects. New techniques that provide automated, standardized, and data-driven techniques are needed.

2.2 Structural Health Monitoring

To overcome the limitations of visual inspections, the field of SHM has aligned itself to mitigate life-safety risks and realize cost savings provided by rapid inspection and continuous monitoring. Over the past 30 years, SHM researchers found many different identification techniques to monitor the changing condition of a variety of structures including mechanical, aerospace, and civil structures. Many SHM literature reviews were performed in a variety of contexts (Brownjohn, 2007; Doebling et al., 1996; Farrar and Worden, 2007; Sohn et al., 2004). This section will not try to recreate these works but instead summarize the common approaches of SHM and indicate some of the draw-backs.

Farrar and Worden (2007) provide an inter-disciplinary description which defines SHM in terms of a four-step statistical pattern recognition paradigm. The four-step process includes:

1. Operational Evaluation

Operational evaluation begins to set the limitations on what will be monitored and how the monitoring will be accomplished.

2. Data Acquisition

The data acquisition portion involves selection of the excitation methods; sensor types and locations; and the network transmission and data storage hardware. These selections often create limitations for SHM implementations and should be considered when developing new procedures.

3. Feature Extraction and Information Condensation

This portion receives the most attention in the literature and has a significant effect on the preceding steps. Common features selected are stiffness and modal characteristics. This step is important in reducing the data to a manageable size that can be reviewed by the engineer.

4. Statistical Model Development

To discriminate between the undamaged and damaged states, it is necessary to develop a statistical model that can analyze the statistical distributions of the measured features. This step is concerned with finding the existence, location, type, and extent of damage. Additionally, a prognosis of the health of the structure can be provided.

In this study, the author will be primarily concerned with feature extraction and statistical model development. However, much of the work relies on assumptions for operational evaluation and data acquisition. As such, this study will concern itself with techniques that estimate the stiffness using acceleration signals at sparse locations. Several global SHM techniques that use similar assumptions are described herein.

Caicedo et al. (2004) describe an application of the eigensystem realization algorithm (ERA) method (Juang and Pappa, 1985) to Phase I of the IASC-ASCE Benchmark SHM problem. Their analysis utilized a plane-frame ROM of the three-dimensional, two-bay by two-bay, four story testbed structure. Then, assuming unknown inputs, the natural excitation technique (NeXT) technique is exploited, where cross-correlation functions are used instead of time histories in the ERA formulation. The natural frequencies, mode shapes, and stiffness are computed through a least squares (LS) procedure and applied to a variety of damage cases. Multiple cases are considered, including time histories generated with the full order model, asymmetric mass causing lateral-torsional coupling, and limited sensor information. In most cases, the authors found that the method was capable of predicting stiffness within 1%.

Sim et al. (2010) present a technique to use a network of distributed wireless sensors to combine local modal information and identify global modes. This method is implemented by considering local groups of sensors and using them to compute the local modal information. Then, global modes are computed using ERA and NeXT and the global modes are combined using a LS estimation procedure. In the case of measurement noise, they developed a technique to reject computed modes of a local group if they do not correlate with the other computed modes. Interestingly, they report that the first mode was often rejected which suggests that local vibrations do not have sufficient frequency bandwidth to measure low frequency modes.

Lew (1995) presents a method of detecting damage through deviations in the identified transfer function parameters. This paper considers the effects of environmental change where an ensemble of undamaged time histories are identified and the maximum and minimum values for each transfer function parameter are tabulated. He uses ERA to identify the transfer functions using noisy time histories. The study is performed on a ninebay truss structure and damage is simulated by the removal of one beam. Damage was successfully detected as identified transfer function parameters deviating from tabulated intervals.

Kim et al. (2005) present a new method to identify multi-input, multi-output systems in the frequency domain. The method uses experimentally determined frequency response function (FRF) data to identify a rational polynomial transfer function. Identification is accomplished by using physical relations to minimize the number of identified values and a three-part optimization procedure is followed. First, an initial estimate is provided by a linear LS method. Second, the Steiglitz-McBride method is applied to a nonlinear estimator. Third, the Levenberg-Marquardt method is applied to a maximum likelihood (ML) estimator. The method is validated with experimental and simulated data on a smart base-isolated structure employing a magnetorheological damper and a two-story structure employing an active mass driver (AMD). Results indicate that the proposed method is quite effective at identifying transfer function models of common civil structural systems. Damage detection was not directly explored but these methods could be combined with that of Lew (1995), for example, to provide damage detection measures.

Interested readers are referred to several excellent SHM literature reviews to cover the wide range of research styles (Brownjohn, 2007; Doebling et al., 1996; Farrar and Worden, 2007; Sohn et al., 2004).

Global methods are not considered in this study because of the limitations they present in a full-scale implementation. Civil structures contain many degrees of freedom and complicated damage scenarios that often cannot be detected using global methods. Furthermore, implementation of global algorithms require cost-prohibitive sensor and cabling requirements or else present un-realistic data transfer requirements in a wireless smart sensor network. Therefore, this study will look to SHM methods that can be implemented in a decentralized manner.

2.3 Substructure Identification

The last two decades saw a number of researchers using substructure identification to perform SHM on civil structures. This focus is a result of substructure identification's ability to simplify both analysis and data processing. Additionally, as wireless smart sensors mature (Lynch and Loh, 2006), substructure identification offers a clear algorithmic advantage due to its decentralized nature.

Decades before civil engineers were using substructure identification, substructure analysis was an important tool used by aerospace engineers to analyze complex structures. Craig and Bampton (1968), building off the work of Hurty (1965), developed a technique to decompose a complex structure into a series of smaller substructures. This methodology, and the closely related component mode synthesis (CMS), allowed design teams to make local changes to a component and analyze its global effects in an efficient manner using local modal behavior.

The benefit of substructure analysis and CMS is that large, complex structures can be broken into smaller parts that are easier to analyze and subsequently design. This process is directly applicable to the field of SHM because complicated structures that refuse to be confidently identified can be broken apart, *analytically*, and studied in smaller substructures. Moreover, many structures have critical components that are known to be susceptible to damage. Substructure identification allows the analyst to focus identification energies on these components, providing for an analysis that is more damage-sensitive when compared to global identification.

This section will describe the state-of-the-art of substructure identification. It will be broadly organized by first discussing linear parametric substructure models. Then, nonlinear parametric models will be considered and, finally, model-free methods.

2.3.1 Linear Parametric Methods

The majority of substructure identification research has focused on a linear substructure model that can be parametrized by a small set of identified features; many researchers have applied a diverse set of such methods. Some researchers have focused on time domain identification using an extended Kalman filter (EKF), an auto-regressive moving average with exogenous input (ARMAX) model, and so forth. Other researchers have focused on the frequency domain and performed parametric identification on combinations of measured FRFs and cross-power spectral densities (CPSDs). Still others used a Bayesian

methodology or focused on different substructure isolation methods. This section will describe these methods in a damage detection context.

Koh et al. (1991) present the first noted application of substructure identification for a civil structure. This study used an EKF to identify the stiffness and damping parameters of a selected substructure. Substructuring was accomplished by partitioning the global coordinates as internal or interface coordinates and then re-writing the equation of motion (EOM) of the substructure with the interface nodes applying force to the substructure. This selection was not minimal and all of the substructures considered were multiple degree of freedom (MDOF) structures themselves. In particular, a six story shear building was considered with the top three stories and bottom three stories as two substructures. With a limited set of measured acceleration response variables on floors three and six, they found that substructure identification significantly out-performed identification using the complete structural model. The values identified by substructure identification converged rapidly even with poor initial guesses. The work concludes with a frame and truss bridge example.

Tee et al. (2005) provide a first- and second-order model for substructure identification of a shear building. The first-order formulation utilizes the ERA method and observer/Kalman filter identification to identify a substructure's stiffness and damping parameters. The second-order model is based on a LS identification of stiffness and damping parameters using the sum of the square distance between the estimated restoring forces and the inertia and/or applied force. Results are verified via numerical simulation of 12 degree of freedom (DOF) and 50 DOF shear buildings subject to base excitation with additive measurement noise. Experimental studies are performed on a 2 m, 12 DOF shear structure. Statistically significant damage is detected in both numerical and experimental work.

Hou et al. (2011) develop a substructure isolation method using the virtual distortion method. This method adds a virtual fixed support to the substructure which isolates the substructure from the global structure. This allows the analyst to treat the substructure as an independent structure using any system identification. In this study, they use a numerical plane frame truss structure and an experimental continuous beam for verification. In both cases, damage was successfully detected in the presence of measurement noise.

Xie and Mita (2010) describe a technique to use CMS to decompose and identify the dynamics of the superstructure of a base-isolated structure. They represent the superstructure as a truncated set of the superstructure vibration modes, which, in turn, simplifies analysis. They found that this representation was less sensitive to incorrect mass estimates and allowed for satisfactory results with fewer total sensors. The method was validated in simulation and demonstrated experimentally on a full-scale, eight-story base isolated structure. This study was used for system identification but could be adapted for a SHM regime.

Xing and Mita (2012) propose a time domain substructure identification procedure that utilizes an ARMAX model. This study focuses on shear buildings with discrete masses and considers minimal substructures (one DOF). By representing the time derivatives through a difference equation, the EOM is decomposed into a time-delayed equation of the acceleration response of the floor level considered and adjacent floors. This equation is easily demonstrated to be an ARMAX model. The simulation structure considered is a five story shear building with a fundamental frequency of 4.5 Hz. The structure

14

is excited by white noise base excitation and 5% measurement noise is added to the acceleration time histories. Five damage levels are considered where the stiffness of each floor level is decreased by 10–50%. Simulation showed that damage is detected with statistical significance for most damage cases considered and for all of the damage cases corresponding to stiffness loss greater than 20%. Incidentally, damage is detected at the floor below the damaged floor as well. These results were compared to an auto-regressive with exogenous input (ARX) model which was found to perform poorly with high error and variance.

The simulation results of Xing and Mita (2012) were verified through experimental testing on a five-story structure with story-level height of 1 m. Damage was simulated by removing a central column. For each damage case, statistically significant damage was identified in the story level and the one below. Larger error and variance was observed for the third floor identification (especially in the case of first floor damage) though not high enough to effect the statistical significance of damage detection. This behavior is predicted in a similar 5-story shear building simulated by Zhang and Johnson (2012b).

Koh et al. (2003) and Trinh and Koh (2011) provide results of substructure identification of the stiffness parameters of a chain structure identified with genetic algorithms. The substructuring is accomplished with or without overlap and a new technique called progressive substructure identification is introduced. Progressive substructure identification works by gradually expanding the substructure considered to include new unknown, *to be identified*, parameters. Numerical results in Koh et al. (2003) show that progressive substructure identification performs best, followed by substructure identification with no overlap, which outperforms substructure identification with overlap. It is also noted that, as the number of unknowns increases, complete structure identification performs poorly. Trinh and Koh (2011) provide experimental verification using a 2 m, 10 DOF shear building. These results show that substructure identification outperforms complete substructure identification, especially in the case of incomplete measurements.

Yuen and Katafygiotis (2006) present a substructure identification procedure in a Bayesian context. A MDOF substructure is considered and the measured responses are partitioned into two sets. Working in the frequency domain, one set of responses is written in terms of the other through the use of transfer functions and a set of trial identification parameters. This allows computing the probability of one set of measurements conditioned on the other set of measurements and on the identified parametric system model. The optimal parameter values for a particular model are found by maximizing the likelihood of this conditional probability. The study concluded with numerical simulation of a 100 DOF structure, successfully detecting damage as small as 5% stiffness loss in one floor. The substructure considered was the lower five stories.

Zhao et al. (1995) develop a LS estimator for frequency domain identification of stiffness parameters for a shear building structure. Substructures were considered as combinations of multiple floors. The results are performed for various levels of additive white noise on the simulated time histories. The substructure identification procedure out-performed complete structure identification.

Zhang and Johnson perform substructure identification in the frequency domain using a minimal substructure model. A nonlinear least squares estimator of the floor-level stiffness and damping parameters is used. The estimator is specifically formed to ensure that the substructure behaves as a single degree of freedom (SDOF) oscillator, which greatly simplifies the dynamics. Zhang and Johnson (2012a) uses estimated FRFs of the floor-level acceleration at, below, and above the floor being analyzed. One of the acceleration time histories is selected as the FRF input to ensure minimum noise effects. Zhang and Johnson (2012c) presents an estimator formed with the cross power spectral densities of the floor-level acceleration below, at, or above the floor being analyzed. The reference signal is specifically chosen to minimize the effects of measurement noise.

In both studies, numerical simulation is performed on a uniform shear structure subject to base excitation. Additive measurement noise is considered, up to 50% root mean square (RMS) in Zhang and Johnson (2012c). In both studies, Monte Carlo simulation is used to determine estimator performance and small levels of error variance are observed for identified stiffness parameters. This indicates that substructure identification is wellsuited for damage detection.

Both studies perform a first-order, approximate analytical error analysis to predict which substructures will be identified with less error than others. For a five story shear building, third story substructure identification provides the worst identification performance. This analysis is exploited in Zhang and Johnson (2012b) to temporarily re-purpose a structural control device to improve identification accuracy. This is demonstrated in simulation using an AMD to temporarily improve the identification accuracy of the third floor stiffness and damping. This control paradigm is also used in Zhang et al. (2009, 2010) to improve the identification accuracy of a 1 m, two-story, flexible shear structure. The details are described further in Chapter 8.

2.3.2 Nonlinear Parametric Methods

In contrast with linear parametric substructure methods, nonlinear parametric substructure methods do not rely on a governing linear model for the substructure's dynamics, which allows the consideration of more complicated models. One common nonlinear model that approximates hysteretic behavior is the Bouc-Wen model (Wen, 1976). This particular form of non-linearity is discussed in Smyth et al. (1999) and Yang and Lin (2005). A more general form of accounting for non-linearity in the substructure is presented in Hernandez-Garcia et al. (2010) which uses Chebyshev polynomials to represent nonlinear behavior in the displacement and velocity terms. Finally, Koh and Shankar (2003) present a procedure that can be adapted to consider non-linearities in the parameters models with arbitrary frequency dependence. Many studies researched identification of nonlinear systems; however, this review will consider only those methods capable of decentralized implementation.

Smyth et al. (1999) develop an on-line parametric identification procedure for nonlinear systems. The procedure relies on a Bouc-Wen representation of the element level non-linearity. The problem is then linearized by representing the Bouc-Wen non-linearity as a truncated series which becomes linear in the parameters. The parameters of this approximated model are then identified using a modified LS adaptive law with a forgetting factor. This allows the determination of the system parameters at each time step to instantaneously detect changes that can signify damage. This method is applied to SDOF and MDOF systems with both over- and under-parametrized models. This method is easily implemented in a decentralized manner by considering interface forces.

Yang and Lin (2005) develop an adaptive tracking parameter identification procedure for linear and nonlinear systems. This method is similar in derivation to Smyth et al. (1999) which follows a LS procedure. In contrast, Yang and Lin choose the adaptive matrix through the minimization of the weighted absolute value between sequential parameter estimates. They find that this method provides tighter tracking to abrupt parameter changes than an adaptive tracking with a forgetting factor. Linear and nonlinear systems are considered though non-linearities are restricted to a cubic displacement term.

Hernandez-Garcia et al. (2010) present a model-free method of identifying the nonlinear restoring forces in a chain structure. This method uses a truncated series of Chebyshev polynomials of the story displacement and velocity. This generalized formulation is used to capture the dominant nonlinear features of the restoring force at a particular connection. The identified model can then be used to identify damage by tracking changes to the Chebyshev series coefficients. This method was experimentally verified on a three-story, aluminum, shear building subjected to white noise excitation. Non-linearities were introduced through a gap between braces at a particular floor level; in the case of no introduced non-linearity, the floor-level restoring force expansion simplified to linear stiffness and damping forces. Results showed a statistically significant change in cases of damage.

Koh and Shankar (2003) present a method to identify a substructure's parameter without measuring interface forces. The method relies on a dense measurement of the substructure's response, which is broken into two or more sets that are used to predict the interface force based on the unknown, *to be identified*, parameters. The distance between the estimated parameters is used as the fitness function and genetic algorithms are used to compute the optimal parameter value. Several different continuous beams are tested and

19

successfully identified in the presence of additive white noise (5% RMS). The stiffness of a 10 story substructure of a 50 story shear building is also identified. This method, though presented using a linear model, can easily be adapted to account for a non-linearity in the parameters with arbitrary frequency dependence.

2.3.3 Model-free Methods

Separate from linear and nonlinear substructure methods, model-free methods can detect damage without an underlying physical model. While not a proper substructure method, model-free methods can detect local damage in a decentralized fashion, which make them important analogues to substructure identification methods. In this section, the use of information theory to detection damage is described and two applications are discussed.

Nichols et al. (2006a) present two information-theoretic measures for use in detecting non-linearities in structures. The first measure is the time-delayed mutual information, which is the distance from the hypothesis of statistical independence for a particular time delay. The second measure is the time-delayed transfer entropy, which is the distance from the hypothesis that dynamics can be described entirely by past history. Both of these measures provide a scalar value for two points within a structure at a particular time-delay. Using these measures, it is shown that non-linearities in the connectivity of a chain structure can be detected using either measure. This method is model-free and distributed because it only relies on two measured time histories and is able to detect nonlinearites within the frequency bandwidth measured. Neither measure relies on a particular structural model nor assumed distribution as the probability density function (pdf) is estimated using a kernel estimator. Of interest to SHM researchers, these measures remain invariant to global *linear* changes to the structure as may be experienced during changing environmental conditions.

Nichols et al. (2006b) present a study that utilizes both time-delayed mutual information and transfer entropy to detect impact damage in composite structures. A composite plate and unmanned aerial vehicle (UAV) wing were instrumented with fiber Bragg grating strain sensors and excited with white noise excitation. Several damage cases were considered with impact damage of various energies. They found that the two measures were able to detect damage-induced non-linearities independent of a baseline, confirming the information theoretic measures as absolute measures of non-linearity.

Overbey and Todd (2009) utilize the time-delayed transfer entropy to characterize damage analytically and experimentally in a frame structure. They found that damage detection is possible under low input signal to noise ratio (SNR) but not possible with output SNR under 30 dB. This behavior was observed in simulation and experiment. Further experiments showed that preload loss in bolted connections was detected when the output SNR was 60 dB.

2.4 Limitations of Current Substructure Identification

This section will describe several limitations of current research in substructure identification. While this analysis is directed towards substructure identification (as is the focus of this study), many of the criticisms will be valid for SHM research as well.

The fundamental question that all SHM researchers should be asking is: To what extent does structural damage cause a detectable change in identified features? This question directs attention to several areas which will be discussed herein. First, how can common structural damage scenarios be modeled and detected? Second, do higher-order dynamics, present in physical systems, contribute to identification error? Third, what are the prerequisites for identifying features; specifically, what levels of excitation and response are required? Finally, what methods are in place for quantifying uncertainty and predicting statistically confident damage prediction in the presence of perturbing effects?

Damage is a complex process and affects structures on many different scales. At the primary level, damage is initiated by cracks and material imperfections in structural components. As the damage progresses, the component properties are degraded, which can ultimately cause system-level changes. Absent an atomistic model, each identification program is an approximation of a physical system. Therefore, successful identification models will contain features that are susceptible to the component and system level changes that real damage causes. Specific investigation of the effect of degraded sections, cracked joints and others is necessary to determine if damage makes discernible changes in the identified features.

A properly identified system that successfully approximates system dynamics to a given precision may be incapable of identifying certain types of damage. For example, a structure that admits a vibration node at the third floor will not detect damage there by tracking modal properties. Successful SHM researchers will do well to move from a system identification-centric mindset to one that approaches damage detection from a feature-extraction perspective.

In the realm of substructure identification, many researchers are concerned with identifying the totality of the structure at the price of detecting damage features. This misses a significant advantage of substructure identification that substructure identification is

22

capable of identifying a component in isolation of the rest of the structure. Furthermore, it is possible to find that a feature that may not represent the "true" structural parameter but remains sensitive to damage. In other words, an identified feature that has a consistent bias error with low error variance is preferable to one that has no bias but larger variance. In either case, it is important that the identified feature change in a statistically significant way for considered damage scenarios.

In addition to considering how damage creates detectable changes in identified features, it is necessary to quantify how higher-order and unmodeled dynamics effect identification. Many researchers use the same ROM to generate simulation data and to perform identification (this author included), which is not ideal and can often obscure shortcomings of a particular method. It is precisely because this practice creates a one-to-one inverse problem that it should be avoided.

Specifically within substructure identification, it is important to separate an analysis model from the simulation model. This is accomplished by using a higher-order model to generate data and then performing substructure analysis using a lower-order model. In addition, this can be accomplished in the laboratory using a physical model that is not unduly restrained to the dynamics of the analysis model.

Another important consideration is how physical limitations and other implementation issues affect the assumptions prerequisite to an identification model. For instance, many researchers consider a structure that is subject to ambient vibrations but then allow their simulations to use high SNRs, stationary noise, and zero unmodeled disturbances. In a more extreme example, some researchers assume noise-free full-state measurement.
These assumptions can easily violate physical realities, even with generous measurement conditions.

To this end, substructure identification can overcome unmodeled disturbances by considering a small area for analysis. In such a case, disturbances outside the analysis area will not directly effect identification. However, it is still necessary to ask important questions about the excitation including: What minimum levels are required to generate measurable response and how much noise is likely to be present?

The final consideration, and arguably the most important for damage detection, is the level of statistical confidence in identified features. Many researchers neglect to present a statistical test for determining the confidence of identified features. This test is necessary to create a hypothesis testing framework to offer a damage diagnosis. Furthermore, it is necessary to analyze the performance of the hypothesis test through a range of normal operating conditions and damage scenarios to determine the level of Type I and II error¹ for a given identification regime.

Substructure identification's damage detection performance is characterized by introducing damage to a substructure and then identifying both that substructure and adjacent undamaged substructures. By comparing these results to an undamaged baseline, it is possible to evaluate if statistically significant damage has been detected and localized correctly.

In conclusion, substructure identification, as currently studied, needs to move away from a system identification paradigm and approach damage detection as a statistical

¹Type I and II errors are colloquially known as false positives and false negatives, respectively.

hypothesis test. This will enable consideration of a variety of implementation issues that will speed the adoption of important SHM methods.

Chapter 3

Testbed Structure

This chapter describes a testbed structure that will be used throughout this document in various simulations. A uniform testbed allows for consistent demonstration of different techniques and a controlled comparison of different methods. Moreover, the focus of later sections will not be interrupted with a description of the testbed structure.¹

During the first reading of this chapter, it may not be clear why certain details are included. Many of the details are specified as a result of previous studies or motivated by theoretical and numerical results. It is the author's hope that any unclear description here will be ameliorated by further discussion in later chapters. When possible, specific references will be made to cross-reference default parameters in this chapter with their corresponding theoretical development and numerical use in later chapters.

In Section 3.1, the testbed structure will be introduced and parameter uncertainty will be analyzed. Modal properties will be computed. The structure will be subject to base excitation and the input signal will be characterized. Finally, the numerical implementation is described.

¹Other simulations and experimental structures will be considered in later chapters.

In Section 3.2, the response measurements will be detailed. In this study, acceleration is the only measured response; its characteristics will be described. Practical implementation issues, including signal to noise ratio (SNR), are explained.

Finally, Section 3.3 concludes with a list of simulation parameters that govern signal processing. This section contains default values and a description for each.

3.1 Structure

The testbed structure is a uniform chain structure with linear stiffness and damping and discrete lumped mass. This model is commonly chosen for structural analysis for its simplicity and general applicability. Moreover, it is often used to analyze civil structures that behave as a shear building. A schematic is shown in Figure 3.1.

In this study, the stiffness and damping parameters are introduced as mass-normalized quantities and chosen such that the natural frequencies and damping ratios are within the range of common civil structures. The story-level normalized stiffness parameter is 1833 s^{-2} and the normalized damping parameter is 8.53 s^{-1} , corresponding to an interstory natural frequency of 6.81 Hz and damping ratio of 10%. The selected damping is toward the higher end of the range of commonly encountered damping ratios for civil structures because better numerical performance is found in simulation. The normalized mass parameter is unity. Using these parameters, the first five natural frequencies and mode shapes are shown in Figure 3.2.

The excitation is taken as ground motion and is modeled as filtered white noise. The particular filter used is a 5th order low-pass butterworth filter with a 20 Hz cutoff frequency. For each simulation, 10 minutes of a filtered white noise time history is used as the input



Figure 3.1: Testbed structure showing the lumped mass model with identified floor highlighted.

signal to a state-space model of the structure; the acceleration response is found using the MATLAB command lsim. When only a particular substructure is being identified, the output is restricted to those adjacent floor-levels to provide extra computational efficiency.

For each simulation, the true structural parameters are used to generate the time histories. The only exception is when a particular damage case is being simulated, in which case the damaged parameters are used to generate the time history. During analysis, an *a priori* guess of the parameters is used to start deterministic optimization. In this case, uncertainty is included by assuming a uniform random variable, with a 2% bias error and a 10% range, for the stiffness of the story being identified. By including a bias in the initial guess, it is easy to determine if the optimizer is simply converging to the initial conditions.



Figure 3.2: The first five natural frequencies [Hz] with modal damping ratios [%] and mode shapes for the testbed structure.

Likewise, the stiffness and damping parameters of the story above the identified story (needed for identifying a particular story) are modeled as Gaussian random variables with zero mean error and 5% coefficient of variation. In the first case, the uncertainty is included by sampling from these distributions for each simulation.

As shown in Figure 3.1, the third floor is the one to be identified, which will be taken as the primary testbed. This story is chosen because it has middling performance in substructure identification and is not one of the boundary cases. This provides a good approximation of the performance when only one floor will be simulated. The substructure is shown by itself in Figure 3.3, which illustrates that only three measured responses are needed for analysis.



Figure 3.3: Substructure with labeled parameters and acceleration response.

3.2 Response

The measured response is the floor-level horizontal absolute acceleration as provided by accelerometers. To simulate limitations encountered in the field, the acceleration response is polluted with additive white noise. The level of noise added is given by the signal to noise ratio (SNR), which is the ratio of the noise-free signal magnitude to the noise magnitude. In this study, SNR is defined in terms of the standard deviation of the signals,

$$SNR = \frac{\sigma_u}{\sigma_n} \tag{3.1}$$

where σ_u is the standard deviation of the noise-free signal u and σ_n is the standard deviation of the noise n such that the measured signal is $\hat{u} = u + n$. SNR is often described in decibels, defined herein as $20 \log_{10} \sigma_u / \sigma_n$.

For each simulation, the acceleration response is generated as described previously and then polluted with additive white noise generated from a sampled Gaussian distribution and scaled to achieve the appropriate SNR. The same SNR is used for all measured responses in a simulation. Different absolute noise magnitude is used on different responses due to the differing levels of response magnitude throughout the structure. This assumption changes slightly in Chapter 7; details are described there.

In this study, the default SNR is 30 dB which is roughly equivalent to noise magnitude that is 3% of the signal's magnitude. This value is selected to match published noise levels of commonly used wireless sensor platforms (Rice, 2010) and assumes that the base excitation has a peak ground acceleration (PGA) of approximately 0.1 m/s^2 . This excitation can be provided by ambient sources or supplemented with a small mechanical shaker installed at the ground level.

3.3 Signal Processing

Prior to analysis, the simulated response signals need to be processed. This step relies on several different parameters governed by the selected data processing procedure. Specific techniques are described in Section 4.3.2; the parameters are listed here.

• Frequency Range $(0.8\omega_{0,i} \text{ to } 1.2\omega_{0,i})$:

The frequency range determines which frequency values will be used for nonlinear regression. The range is selected so that there is sufficient magnitude in the response of the model function H_{MOD} and is limited so as to not include spurious noise at frequencies of low response. The default value is 0.8–1.2 of the substructure natural

frequency, $\omega_{0,i} = \sqrt{k_i/m_i}$. The number of frequency points in this range is called the statistical degrees of freedom.

• Number of Ensemble Records (*M* = 42):

The number of ensemble records is dictated by the time duration, NFFT, signal overlap, and sample rate. A large number of ensemble records ensures that the FRF estimate converges.

• **NFFT** ($N_{\text{FFT}}=2^{12}$):

The number of points used in the discrete Fourier transform (DFT), along with the sample rate, determines the frequency resolution of the estimated FRFs. A power of 2 is used to achieve the computational performance gain while using the fast Fourier transform (FFT) algorithm.

• **Sample Rate** (1 / *T_s*=100 Hz):

The sample rate is the frequency at which data points are collected in the time history. 100 Hz is chosen to conservatively keep the entire frequency bandwidth of the structure below the Nyquist frequency.

• **SNR** (30 dB):

The SNR is defined in Section 3.2.

• Signal Overlap (ρ =67%):

The signal overlap is the fraction of overlap of time domain data between neighboring ensembles. Antoni and Schoukens (2007) provide the optimal selection of signal overlap. • Statistical Degrees of Freedom (*N* = 112):

The statistical degrees of freedom are determined by the frequency range and NFFT. The default value is 112, which is the number of different points considered in each regression. A large number of points ensures convergence and provides for decreased statistical curvature.

• **Time Duration** (10 minutes):

The time duration determines how much data is collected for each simulation. Combined with the sample rate, this specification dictates that 60,000 data points will be generated for each time history.

• Window Function ($w(t) = sin(\pi t / N_{FFT}T_s)$):

The half-range sine function is used to window signals while constructing an ensemble. Antoni and Schoukens (2007) provide the optimal selection of window function.

3.4 Monte Carlo Simulation

This study uses Monte Carlo simulation (MCS) to develop statistically significant identification performance statistics. Unfortunately, the distribution of the identified parameter (in this study, the story stiffness) is not known *a priori*. Therefore, it is necessary to perform many independent identification experiments to determine the performance. In this study, 10,000 independent experiments will be used to determine the distribution and statistics of the identified parameters. This section will describe the confidence levels that can be ascribed to statistics associated with the simulation.



Figure 3.4: Identified stiffness parameter distribution shown as a histogram and QQ-plot for 10,000 samples. The identified parameter is the third story stiffness of the testbed structure and is normalized by the true value.

After performing 10,000 independent identification experiments, the distribution of the identified parameter is shown in Figure 3.4. By inspection, it is clear that the identified parameter assumes a Gaussian distribution. This is confirmed by the Lilliefors test which accepts the null hypothesis that the distribution is Gaussian (Lilliefors, 1967).

Now that the identified parameter distribution is confirmed to be Gaussian, confidence intervals of the sample mean and variance can be developed. For a Gaussian random variable with unknown mean and variance, the sample mean is an unbiased estimator of the mean and the sample distribution is given by Student's T distribution (Leon-Garcia, 2007). The confidence interval of the sample mean \overline{X}_n is,

$$CI_{\alpha}(\overline{X}_n) := \left[\overline{X}_n - t_{\alpha/2, n-1}\hat{\sigma}_n/\sqrt{n}, \overline{X}_n + t_{\alpha/2, n-1}\hat{\sigma}_n/\sqrt{n}\right]$$

where α is the confidence level, $\hat{\sigma}_n$ is the sample standard deviation, and n is the number of independent experiments. In this study, the sample mean confidence interval was routinely below a hundreth of one percent of the estimated sample mean.

The sample variance distribution $\hat{\sigma}_n^2$ of a Gaussian random variable of unknown mean and variance is a χ^2 random variable (Leon-Garcia, 2007). The confidence interval is,

$$CI_{\alpha}(\hat{\sigma}_n^2) := \left[\frac{(n-1)\hat{\sigma}_n^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)\hat{\sigma}_n^2}{\chi^2_{1-\alpha/2,n-1}}\right]$$

In this study, the sample variance confidence interval was routinely less than 3% of the estimated sample variance. This implies that the sample standard deviation confidence interval will be below 1.5% of the estimated sample standard deviation.

By using a large number of independent identification experiments, statistical signficant results are found for the mean and variance of each identified parameter. Part I

Theoretical Developments

Chapter 4

Substructure Identification Estimator

This chapter describes the development of a substructure identification estimator. The estimator contains two components, a model function and a function of estimated quantities. The first section will describe the development of these two functions. The second section will describe various nonlinear function estimation procedures to estimate the first two statistical moments of the function of estimated quantities. The third section will demonstrate how substructure identification can be used for decentralized processing within a network of smart sensors. The final section will develop an error prediction function to predict which structure–substructure combinations will admit higher identification error.

4.1 Estimator Formulation

This section describes the formulation of a substructure identification estimator. The estimator is characterized by a model function and a function of estimated quantities that map the identification parameters and estimated quantities, respectively. These two functions are selected based on the reduced order model (ROM) and equation of motion (EOM) that govern the structure's dynamics. The substructure estimator is not

uniquely defined and can be formed in different configurations. This section will describe how a substructure estimator is derived and give an example for the shear building testbed.

First, ROMs are discussed including three different ways to form a ROM for a given structure. Second, the model function is introduced and a selection for the testbed problem is provided. Third, the EOM is presented within the context of a ROM. Fourth, the substructure estimator is formed for the shear building testbed. Finally, a discussion of the various properties of substructure identification estimators is provided.

4.1.1 Reduced Order Model

Substructure identification relies on a reduced order model (ROM) describing a portion of a structural system. If the structure is continuous, the ROM can be considered as a coarse finite element model. For a discrete structure, the ROM is simply the elements of a portion of the structure. Once the ROM is developed, substructure identification works by isolating a portion of the structure to identify the stiffness of that substructure. This stiffness value is then used to infer damage in a long term monitoring program or post-event evaluation.

There are three basic methods to form a ROM and the accompanying substructure representation: component mode synthesis (CMS), static condensation, or a direct finite element model. These three methods create a ROM representation of the structure that characterizes the dynamics of the structure and then, define the behavior of each DOF in terms of a small set of adjacent DOFs. These methods can often result in the same ROM and should be chosen by the analyst to simplify the procedure. The first method discussed is CMS. Much of what guides substructure identification was developed by early pioneers in CMS who developed techniques to model a portion of the structure independent of the global structure. Craig and Bampton (1968) introduced a refinement of CMS that allows the analyst to describe a substructure in terms of its normal modes and constraint modes (formed by its connectivity to the rest of the structure). Their analysis computed the modes of the substructure and then reduced the order of these modes to form reduced-order stiffness and mass matrices. The reduced order matrices were re-combined to form global stiffness and mass matrices. They found that the global modes computed from the ROM had excellent agreement with the full-order structural model. This method can be used to compute a reduced set of substructure modes that can be identified using only the internal and interface DOFs. The identified modes can be tracked for damage detection or monitoring. Moreover, the results of Craig and Bampton (1968) show that, when taken as a whole, the ROM is an adequate representation of the global dynamics of the structure.

The second method discussed herein is static condensation (Chopra, 2001). Static condensation works by removing mass-less DOFs from the model. This is accomplished by partitioning the mass and stiffness matrices and then solving for the mass-less DOFs in terms of the DOFs with mass. These are then combined to form a new reduced stiffness matrix. Structural models rarely contain mass-less DOFs, which means that the static condensation process needs to be started by first removing the mass from a set of DOFs. This is accomplished using engineering judgment to lump the mass from adjacent DOFs into one resultant DOF. Using the ROM generated from static condensation for substructure identification is straight-forward as each mass-containing DOF can be treated as an independent substructure.

The final method, herein, is using a coarse finite element model to describe the structure. This method works well on continuous structures such as plates and beams because it simply forms a finite element model with a small number of elements rather than a large set. The same stiffness relationships are used to describe the connectivity; a single node can be analyzed as a substructure.

As stated previously, these methods do not need to be used in exclusion to one another and often give the same results. CMS is best suited to complicated structures with non-uniform structural components whereas static condensation and coarse finite element models are best suited for regular structures with uniform structural components. Regardless of the method used, the primary importance is to generate a low order model that can be broken up into substructures.

4.1.2 Model Function

The model function, H_{MOD} , is a reduced order representation of the local behavior of the substructure. As such, the model function is dependent on the ROM selected for the identified structure. The only requirement for H_{MOD} is that it contains the identification parameter(s), which is often taken to be the substructure stiffness (including in this study). However, this not a unique selection and could just as easily be a natural frequency or non-physical parameter. The model function choice will dictate what identification parameter is used for damage detection. A natural choice for H_{MOD} is a SDOF oscillator with unknown parameters because it simplifies and removes unwanted dynamics. This is not a unique selection but it has been found to have significant benefit when compared to other model functions. For instance, an estimator linear in its parameters is possible but not advantageous because it is an unstable system which amplifies noise effects. Thus, it is beneficial to select a model function that will reduce the order but maximize response for greater detectability.

For the shear building testbed, H_{MOD} is a SDOF oscillator with acceleration output given by the transfer function,

$$H_{\rm MOD}(s) = \frac{1}{1 + \frac{c_i}{m_i s} + \frac{k_i}{m_i s^2}}$$
(4.1)

This model function contains the story-level stiffness k_i , which will be used for damage detection. The story-level damping parameter c_i is also identified though it will not be used for damage detection and is treated as a nuisance regression parameter. The story-level mass m_i is assumed to be known *a priori*, though the identification could be performed by identifying mass normalized stiffness and damping parameters.

4.1.3 Equation of Motion

The next step in forming an estimator for substructure identification is to find an EOM. The EOM can be found using either Newton's 2^{nd} law or Lagrangian mechanics. All that is necessary is the connectivity of the adjacent DOFs. For simple structures with obvious discretizations, Newtonian mechanics is a natural choice. However, for more complicated structures, Lagrangian mechanics can ease analysis. It is assumed that each discrete mass is connected by linear stiffness and linear viscous damping between adjacent DOFs. A linear model is an approximation of the structure. It is reasonable because SHM often relies on the low levels of ambient excitation that generate a linear response in most civil structures. Furthermore, nonlinear damage often results in linear stiffness loss such as a plastic hinge decreasing the lateral stiffness of the frame.

The EOM of the i^{th} floor of a shear building is,

$$m_i \ddot{x}_i + c_i (\dot{x}_i - \dot{x}_{i-1}) + k_i (x_i - x_{i-1}) + c_{i+1} (\dot{x}_i - \dot{x}_{i+1}) + k_{i+1} (x_i - x_{i+1}) = 0$$
(4.2)

where x_i is the displacement of the floor level with respect to an inertial reference frame and over-dots represent the velocity and acceleration, \dot{x}_i and \ddot{x}_i , respectively. In (4.2), there are story level parameters from the story to be identified (m_i , c_i , and k_i) and previous story parameters (c_{i+1} and k_{i+1}).

4.1.4 Shear Building Substructure Estimator

Once the model function is selected, it is necessary to manipulate the EOM to generate an equation that contains the model function. To do this, take the Laplace transform of the EOM and then algebraically manipulate the transformed equation to isolate the chosen model on one side of the equation.

With the shear building model function in mind (4.1), start by taking the Laplace transform of (4.2). Initial conditions can be neglected because signal processing will use a tapered window function. The resulting equation is:

$$m_{i}\ddot{X}_{i}(s) + c_{i}(\dot{X}_{i}(s) - \dot{X}_{i-1}(s)) + k_{i}(X_{i}(s) - X_{i-1}(s)) + c_{i+1}(\dot{X}_{i}(s) - \dot{X}_{i+1}(s)) + k_{i+1}(X_{i}(s) - X_{i+1}(s)) = 0 \quad (4.3)$$

where $X_i(s)$ is the Laplace transform of $x_i(t)$. Re-write the displacement and velocity in terms of the acceleration and add $-m_i \ddot{X}_{i-1}(s)$ to both sides of (4.3). Further algebraic manipulation results in:

$$\frac{1}{1 + \frac{c_i}{m_i s} + \frac{k_i}{m_i s^2}} = \frac{\ddot{X}_i(s) - \ddot{X}_{i-1}(s)}{-\ddot{X}_{i-1}(s) + (\ddot{X}_{i+1}(s) - \ddot{X}_i(s)) \left(\frac{c_{i+1}}{m_i s} + \frac{k_{i+1}}{m_i s^2}\right)}$$
(4.4)

The final step is to rewrite the Laplace transform of the response variables in terms of their transfer functions. In this example, ground motion \ddot{u}_{g} is taken as the excitation (though the excitation could take other forms). As long as the excitation is not directly applied to the substructure being identified, the analysis will simplify as follows in Section 4.3.1.

The Laplace transform of the absolute acceleration response at the *i*th floor level relative to the ground is given by transfer function $H_{\vec{x}_i,\vec{u}_g}(s)$. Thus, (4.4) becomes:

$$\frac{1}{1 + \frac{c_i}{m_i s} + \frac{k_i}{m_i s^2}} = \frac{H_{\ddot{x}_i, \ddot{u}_g}(s) - H_{\ddot{x}_{i-1}, \ddot{u}_g}(s)}{-H_{\ddot{x}_{i-1}, \ddot{u}_g}(s) + (H_{\ddot{x}_{i+1}, \ddot{u}_g}(s) - H_{\ddot{x}_i, \ddot{u}_g}(s)) \left(\frac{c_{i+1}}{m_i s} + \frac{k_{i+1}}{m_i s^2}\right)}$$
(4.5)

By examining (4.5), a substructure identification estimator has been successfully constructed. On the left side is the model function $H_{MOD}(s)$; on the right side is a function that maps the measured response and *a priori* parameters. This function will be denoted the *function of estimated quantities*, $H_{EST}(s)$.

4.1.5 Discussion

The two components of a substructure identification estimator are a model function H_{MOD} and a function of estimated quantities H_{EST} . These functions map the identification parameters and the estimated quantities (measured responses and *a priori* parameters), respectively. The functions describing a substructure identification estimator are based on the selected ROM of the structure but are not uniquely determined. Therefore, it is necessary for the analyst to use judgment to determine the best estimator for a given application.

A useful property of the substructure identification estimator is that it simplifies higher order dynamics of the structure. By selecting a SDOF oscillator to describe the local behavior, the complicated dynamics of the response can be simplified to that of a SDOF system. This is visually demonstrated in Figure 4.1, which shows the inherent simplification of substructure identification within the testbed problem.

A second useful property of a properly constructed substructure identification estimator is that it results in a decentralized algorithm. A decentralized algorithm is an estimator that relies only on local response measurements. The substructure estimator given in (4.4) requires acceleration measurements from the story at, below, and above the story to



Figure 4.1: Function of estimated quantities compared to individual response transfer functions for the testbed structure at the third floor using deterministic transfer functions. Markers are shown in the frequency range where substructure identification takes place.

be identified. This enables implementation in a decentralized network of smart sensors which will be described in Section 4.3.

Finally, substructure identification can provide a means to identify the full structure. If the structure has minimal connectivity between DOFs (*i.e.*, a chain structure), then each stiffness element of the structure can be identified using the previous element's stiffness to determine H_{EST} . For a shear building, the top story is identified using the free surface boundary condition and then, each story is identified using the previous story's identified stiffness and damping parameters. In such a way, the entire structure is identified consistent with the ROM.

4.2 Nonlinear Function Estimation

Before the derived substructure identification estimator can be used in nonlinear regression, the observed data vector must be estimated using H_{EST} as defined in (4.5). H_{EST} is a nonlinear function of the transfer functions of the responses and the previous story parameters. In practice, the computation will use estimates of the FRFs and previous story parameters, which are themselves random variables. Thus, the observed data vector will be an estimate of a nonlinear function of random variables provided by H_{EST} .

This section will describe three techniques to find the observed data vector estimate: linear approximation, the unscented transformation, and Monte Carlo simulation. The section first describes the input random variables and their statistical properties. Then, each method of nonlinear function estimation is presented. Finally, the three methods are compared and a discussion of best practice is provided.

4.2.1 Input Random Variables

The input random variables are the FRFs and previous story parameters used in H_{EST} . The individual input random variables can be represented together in vector notation as $\mathbf{z} = [H_{\vec{x}_{i-1},\vec{u}_g}, H_{\vec{x}_i,\vec{u}_g}, H_{\vec{x}_{i+1},\vec{u}_g}, k_{i+1}, c_{i+1}]^{\mathsf{T}}$. This section describes the statistical properties of \mathbf{z} .

The FRFs in **z** are estimated using Welch's method as described in Section 4.3.2. As a result of using Welch's method, the FRF estimates are random variables distributed as circularly symmetric Gaussian random variables at a particular frequency value (Pintelon and Schoukens, 2001). Likewise, the estimates of the previous story parameters are assumed to be Gaussian random variables so that $\mathbf{z}(j\omega)$ is a Gaussian random vector at a particular frequency value ω . The input random variable is a Gaussian random vector at each frequency value with mean and covariance given by $\boldsymbol{\mu}_{\mathbf{z}}(j\omega)$ and $\mathbf{K}_{\mathbf{z}}(j\omega)$. Each random variable of the input random vector is statistically independent so that $\mathbf{K}_{\mathbf{z}}(j\omega)$ is diagonal.

4.2.2 Linear Approximation

To find the linear approximation for $H_{\text{EST}}(\mathbf{z})$, represent the function as a Taylor series around its expected value $\mu_{\mathbf{z}}$. This series can be truncated to present an approximation of

the original function. In this study, a first order approximation is applied by neglecting the terms of the Taylor series of second and higher order. Thus, $H_{\text{EST}}(\mathbf{z})$ becomes

$$H_{\text{EST}}(\mathbf{z}) = \sum_{i=0}^{\infty} \frac{1}{i!} \frac{\partial^{i} H_{\text{EST}}(\mathbf{z})}{\partial \mathbf{z}^{i}} \Big|_{\mathbf{z}=\mu_{\mathbf{z}}} (\mathbf{z}-\mu_{\mathbf{z}})^{i}$$

$$\approx H_{\text{EST}}(\mu_{\mathbf{z}}) + \frac{\partial H_{\text{EST}}(\mathbf{z})}{\partial \mathbf{z}} \Big|_{\mathbf{z}=\mu_{\mathbf{z}}} (\mathbf{z}-\mu_{\mathbf{z}})$$

$$\approx H_{\text{EST}}(\mu_{\mathbf{z}}) + \mathbf{h}_{\text{EST}}^{\text{H}}(\mathbf{z}-\mu_{\mathbf{z}})$$
(4.6)

where $\mathbf{h}_{\text{EST}}^{\text{H}}$ is the Jacobian evaluated at $\mathbf{z} = \boldsymbol{\mu}_{\mathbf{z}'}$ of $H_{\text{EST}}(\mathbf{z})$ with respect to \mathbf{z} . Note that the notation of (4.6) implies tensor multiplication for higher order terms.

The first and second statistical moments of (4.6) can be approximated as:

$$\mathbb{E}[H_{\text{EST}}(\mathbf{z})] \approx \mathbb{E}\left[H_{\text{EST}}(\boldsymbol{\mu}_{\mathbf{z}}) + \mathbf{h}_{\text{EST}}^{\text{H}}(\mathbf{z} - \boldsymbol{\mu}_{\mathbf{z}})\right]$$

$$\approx H_{\text{EST}}(\boldsymbol{\mu}_{\mathbf{z}})$$
(4.7)

$$\mathbb{E}\left[(H_{\text{EST}}(\mathbf{z}) - \mathbb{E}\left[H_{\text{EST}}(\mathbf{z})\right])^{2}\right] \approx \mathbb{E}\left[\left(H_{\text{EST}}(\boldsymbol{\mu}_{\mathbf{z}}) + \mathbf{h}_{\text{EST}}^{\text{H}}(\mathbf{z} - \boldsymbol{\mu}_{\mathbf{z}}) - H_{\text{EST}}(\boldsymbol{\mu}_{\mathbf{z}})\right)^{2}\right]$$

$$\approx \mathbb{E}\left[\mathbf{h}_{\text{EST}}^{\text{H}}(\mathbf{z} - \boldsymbol{\mu}_{\mathbf{z}})(\mathbf{z} - \boldsymbol{\mu}_{\mathbf{z}})^{\text{H}}\mathbf{h}_{\text{EST}}\right]$$

$$\approx \mathbf{h}_{\text{EST}}^{\text{H}}\mathbf{K}_{\mathbf{z}}\mathbf{h}_{\text{EST}}$$
(4.8)

where $\mathbf{K}_{\mathbf{z}} = \mathbb{E}\left[(\mathbf{z} - \boldsymbol{\mu}_{\mathbf{z}})(\mathbf{z} - \boldsymbol{\mu}_{\mathbf{z}})^{\mathsf{H}} \right]$ is the covariance matrix of the input random variables and $(\cdot)^{\mathsf{H}}$ denotes the complex conjugate transpose. Using (4.7) and (4.8), the mean and covariance of H_{EST} is found. The performance of this method is discussed in Section 4.2.5.

4.2.3 Unscented Transformation

The unscented transformation (UT) is used as a sub-optimal estimator of a nonlinear function of random variables. It was originally developed by Julier and Uhlmann (2004) to linearize Kalman filter equations for nonlinear systems. They describe the underlying assumption of the UT as:

The UT is founded on the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation.

They continue by describing a transformation that propagates a set of sigma points through the nonlinear function. Then the statistics of the output distribution is estimated as a weighted combination of these sigma points. The process is described (Julier and Uhlmann, 2004) as:

- 1. Form a set of sigma points that capture the first two statistical moments of the input random variables.
- 2. Instantiate each point through the nonlinear function to yield the set of transformed sigma points.
- 3. The mean of the output is computed as the weighted average of the transformed points.
- 4. The covariance of the output is computed as the weighted outer product of the transformed points.

In addition to nonlinear Kalman filtering, Julier and Uhlmann (2004) show that the UT is well-suited to the radar tracking problem of polar to Cartesian coordinate conversion. Herein, the UT is used to compute the statistics of H_{EST} using guidelines developed in Merwe (2004) for implementation. The performance of this method is discussed below.

4.2.4 Monte Carlo Simulation

Monte Carlo simulation (MCS) is used to compare the performance of nonlinear function estimation. This procedure works by generating realizations of the input random vector \mathbf{z} and then computing the observed data vector directly using H_{EST} . Then, the computed points are used directly to compute the first two statistical moments of the output $\mathbb{E}[H_{\text{EST}}]$ and $\mathbb{E}[(H_{\text{EST}} - \mathbb{E}[H_{\text{EST}}])^2]$.

4.2.5 Comparison

The statistics of the nonlinear function computed using linear approximation and UT are compared with those computed from MCS. The linear approximation is computed using both a theoretical and numerical Jacobian. The first and second statistical moments of H_{EST} computed with these methods are compared graphically in Figure 4.2 where H_{EST} is estimated and the predicted covariance ellipses are shown for each method.

It is clearly demonstrated that the linear approximation methods and UT have strong agreement in their predicted mean and covariance. The mean value forms the estimate of \hat{H}_{EST} and shows strong agreement with MCS and the true value. Unfortunately, the estimated covariance is an order of magnitude larger that the covariance computed using Monte Carlo methods. By using a large number of Monte Carlo samples (10⁶), statistically significant differences are found. Moreover, MCS is able to better reconstruct the output distribution and should be treated as closely resembling the actual distribution. This result



Figure 4.2: Covariance of the nonlinear function of estimated quantities for various estimation methods. Increasing frequency follows a clockwise path.

is confirmed in Section 5.2.2 where the over-predicted covariance results in meaningless confidence regions in spite of accurate estimates.

This result demonstrates that the distribution of H_{EST} , the nonlinear function of estimated quantities, is not easily estimated and likely non-Gaussian. Both the linear approximation and the UT are unable to construct meaningful estimates of the covariance. Therefore, it is necessary to use regression techniques that do not require second moment information or use *a priori* information about the estimates covariance. This will be discussed further in Chapter 5.

4.3 Decentralized Processing

The substructure identification method is ideal for implementation in a decentralized system. The algorithm itself relies on measured time histories from adjacent floor levels. This trait can be exploited for implementation within a network of wireless smart sensors. This section will describe some of the benefits of decentralized processing. First, the decentralized substructure identification estimator is derived. Next, FRF estimation is described. Then, two possible network topologies are described. Finally, the potential for data reduction is quantified.

4.3.1 Decentralized Substructure Estimator

As derived, the substructure identification estimator relies on deterministic transfer functions from excitation input to floor-level acceleration response $H_{\ddot{x}_i, \ddot{u}_g}(s)$. This information relies on measured input signals that may be difficult to achieve in practice. For the testbed structure, this is achievable by measuring the ground motion but this precludes decentralized identification. However, the substructure estimator can be modified to be written in terms of FRFs between adjacent floor levels. This quantity can be easily estimated with the measured acceleration response from nearby floor-levels.

The first step is to rewrite (4.5) in terms of the FRFs by evaluating the transfer function along the imaginary axis, $H_{\ddot{x}_i, \ddot{u}_g}(s)|_{s=j\omega} = H_{\ddot{x}_i, \ddot{u}_g}(j\omega)$. Then, using the relationship $H_{\ddot{x}_i, \ddot{x}_{i-1}}(j\omega) = H_{\ddot{x}_i, \ddot{u}_g}(j\omega)/H_{\ddot{x}_{i-1}, \ddot{u}_g}(j\omega)$, rewrite (4.5) in terms of the interstory FRFs. Note that, by choosing which floor level is the input, (4.5) can be written in different ways. For a chain structure, the three ways are:

$$H_{\text{EST}}^{(\ddot{x}_{i-1})}(H_{\ddot{x}_{i},\ddot{x}_{i-1}},H_{\ddot{x}_{i+1},\ddot{x}_{i-1}},k_{i+1},c_{i+1},\omega) = \frac{1-H_{\ddot{x}_{i},\ddot{x}_{i-1}}}{1+(H_{\ddot{x}_{i+1},\ddot{x}_{i-1}}-H_{\ddot{x}_{i},\ddot{x}_{i-1}})\left(\frac{jc_{i+1}}{m_{i}\omega}+\frac{k_{i+1}}{m_{i}\omega^{2}}\right)}$$
(4.9a)

$$H_{\text{EST}}^{(\ddot{x}_{i})}(H_{\ddot{x}_{i-1},\ddot{x}_{i}}, H_{\ddot{x}_{i+1},\ddot{x}_{i}}, k_{i+1}, c_{i+1}, \omega) = \frac{H_{\ddot{x}_{i-1},\ddot{x}_{i}} - 1}{H_{\ddot{x}_{i-1},\ddot{x}_{i}} + (H_{\ddot{x}_{i+1},\ddot{x}_{i}} - 1)\left(\frac{jc_{i+1}}{m_{i}\omega} + \frac{k_{i+1}}{m_{i}\omega^{2}}\right)}$$
(4.9b)

$$H_{\text{EST}}^{(\ddot{x}_{i+1})}(H_{\ddot{x}_{i-1},\ddot{x}_{i+1}},H_{\ddot{x}_{i},\ddot{x}_{i+1}},k_{i+1},c_{i+1},\omega) = \frac{H_{\ddot{x}_{i-1},\ddot{x}_{i+1}}-H_{\ddot{x}_{i},\ddot{x}_{i+1}}}{H_{\ddot{x}_{i-1},\ddot{x}_{i+1}}+(1-H_{\ddot{x}_{i},\ddot{x}_{i+1}})\left(\frac{jc_{i+1}}{m_{i}\omega}+\frac{k_{i+1}}{m_{i}\omega^{2}}\right)}$$
(4.9c)

where $H_{\text{EST}}^{(\vec{x}_i)}$ is H_{EST} with the *i*th floor chosen as the input.

By formulating the FRF substructure estimator in three separate ways, the estimator can be implemented to use the most accurate formulation at each frequency value. This is implemented by selecting the signal with the highest power at a particular frequency, which corresponds to higher SNR resulting in improved estimator performance.

4.3.2 Frequency Response Function Estimation

A wide body of signal processing literature is available and will be used to find optimal signal processing techniques to characterize the FRF estimate. SHM is a challenging

signal processing environment that lacks many of the ideal conditions found in the laboratory. Among the challenges are low SNR and random excitation sources with poor statistical properties including non-Gaussian signals, non-stationary signals, and others. To overcome these difficulties, long time records are collected and combined using ensemble averaging. For FRF estimation, this process, called Welch's method (Welch, 1967), is performed by splitting the input and output signals into ensembles of equal length sections, multiplying each section by a window function, taking the DFT, and then, combining the DFTs in the frequency domain. To estimate $\hat{H}_{\vec{x}_i,\vec{x}_{i-1}}(j\omega)$, the FRF from \vec{x}_{i-1} to \vec{x}_i , the following steps are performed (Pintelon and Schoukens, 2001):

1. Section the signals into an ensemble of equal length sections.

$$\tilde{x}_{i}^{(m)}(t) = \begin{cases} \ddot{x}_{i}(t+m(1-\rho)T), & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$
(4.10)

where *T* equals the section time length, ρ is the overlap, and *m* equals the ensemble record index.

2. Consider discrete time points and multiply each section by the window function.

$$\ddot{x}_{i}^{(m)}[n] = w(nT_{\rm s})\tilde{x}_{i}^{(m)}(nT_{\rm s})$$
 for $n = 0, 1, 2, ..., N_{\rm FFT} - 1$ (4.11)

where T_s is the sampling time, w(t) is the window function, and $N_{FFT} = T/T_s$ is the number of samples in each section, which is also the number of DFT points used.

3. Take the discrete Fourier transform of each ensemble record.

$$\ddot{X}_{i}^{(m)}[k] = \sum_{n=0}^{N_{\rm FFT}-1} \ddot{x}_{i}^{(m)}[n] e^{-j2\pi nk/N_{\rm FFT}} \quad \text{for } k = 0, 1, 2, ..., N_{\rm FFT} - 1$$
(4.12)

4. Combine the ensemble to form the FRF estimate.

$$\hat{H}_{\ddot{x}_{i},\ddot{x}_{i-1}}(j\omega_{k}) = \sum_{m=1}^{M} \frac{\left(\ddot{X}_{i-1}^{(m)}[k]\right)^{*} \ddot{X}_{i}^{(m)}[k]}{\left(\ddot{X}_{i-1}^{(m)}[k]\right)^{*} \ddot{X}_{i-1}^{(m)}[k]}$$
(4.13)

where $(\cdot)^*$ denotes the complex conjugate and $\omega_k = 2\pi k/N_{\text{FFT}}$.

 $\hat{H}_{\dot{x}_i,\dot{x}_{i-1}}(j\omega_k)$ is a biased estimator of the FRF and as a result of the Central Limit Theorem, is distributed as a circular symmetric complex Gaussian random variable when a sufficiently large number of ensemble sections are used. Antoni and Schoukens (2007) performed a detailed study to find the best performing window function for FRF estimation. They found that a half-range sine window function with 67% overlap has the lowest bias and error variance for FRF estimates using Welch's method. Herein, these parameters are used as best practice when computing FRF estimates.

In this study, methods of FRF estimation are implemented with custom MATLAB functions using the fft command. This suite of functions outperforms the cpsd command in computational speed by exploiting the limited frequency range of interest.

4.3.3 Network Topologies

Wireless smart sensors have the capability to revolutionize civil SHM. By removing the cabling requirement, wireless sensors offer a flexible, economic solution to instrument a



Figure 4.3: Network topology showing typical sensor nodes in light blue and master nodes in darker blue.

civil structure. Moreover, by including a processor on the sensor board, data processing can be performed in a decentralized manner.

Substructure identification is ideally suited for a decentralized implementation in a network of wireless smart sensors. This section will describe two possible network topologies. In the following section, the potential data transmission reduction will be computed for these two topologies. It is assumed herein that a network of wireless sensors are available such that at least one sensor is installed per floor and acceleration is the measured response. There are two types of network topologies considered: a fully decentralized network and a two-tier network.

- The fully decentralized network (single-tier) works by having each sensor communicate with its directly adjacent neighbors and performs substructure identification locally. This means that the identification for a particular story is performed by the sensor at that story using data from its adjacent neighbors.
- 2. A two-tier network topology is proposed. Each sensor node is equipped with a lowpower, short-range radio used to communicate with nearby nodes. Additionally, on every third floor, the sensor node is replaced with a local master node that has a highpower, long-range radio so that each of the local master nodes can communicate with each other and the user. Alternatively, the network could be arranged so that the local master nodes communicate via a multi-hop network path through the various sensor nodes. See Lynch and Loh (2006) for a summary of the different available network configurations and a discussion of their relative merits.

4.3.4 Data Reduction

In addition to decentralizing the analysis, the substructure identification method has great potential for data reduction, which can provide dramatic power-saving to wireless smart sensors. This is accomplished in two basic steps. First, each time history is transformed to find its FFT, retaining only the points near the substructure's natural frequency used for nonlinear regression. Second, by using a two-tier network topology, repeated data transfers are minimized and the total amount of data transmitted can be further reduced. The first step is performed at the local sensor node. The local sensor node performs data acquisition, ensembling, windowing, and FFT computation. Then a reduced set of FFT points are transmitted for each ensemble. By utilizing a two-tier network, repeated data transfers are eliminated. The local master sensor node aggregates the FFT points, uses them to estimate the relevant FRFs for each substructure, and computes the nonlinear transformation for each substructure. Nonlinear regression is performed to estimate the stiffness and damping parameters for each substructure. These estimates are transmitted to the user who can perform a hypothesis test to determine if the structure is damaged at a particular floor level.

Table 4.1 shows the amount of data reduction possible for the two network topologies. The two-tier network requires only 37% of the total data transmission of the single tier network topology. Moreover, when compared to a fully centralized algorithm that uses the entire time history, the decentralized algorithm reduces the amount of transmitted data by an order of magnitude and with zero fidelity loss.

By designating a local master node every third floor, the total amount of data transferred is reduced by a third. This is a result of adjacent floors requiring access to the same time histories. If this arrangement is not ideal for a particular implementation, the network can be setup as a single tier and the substructure identification is performed locally at each node with zero performance loss and slightly more data transmitted.

In this study, the amount of data transmitted for each network topology is computed using the standard signal processing parameters given in Chapter 3. As such, each sensor will need to transmit 42 ensemble records with 112 FFT points to two different sensors. Each FFT point is a complex number which is represented by two 16-bit floating point

58

	Data transfer [KB] per node				
	Sensor Node		Local Master		
	Tx	Rx	Tx	Rx	Total Tx [KB]
Centralized Network	117.2				3515.6
Single-Tier Network	36.8	36.8			349.1
Two-Tier Network	36.8			73.5	220.5

Table 4.1: Network transmission for various wireless smart sensor network topologies in kilobytes (Tx=transmit, Rx=receive).

numbers. For the centralized topology, the entire time history will be sent to a central server, which means that 60,000 16-bit floating point numbers will be transmitted by each sensor. Depending on the testbed, the amount of data transmitted by each sensor will change but the relative transmission reduction will stay similar.

4.4 Estimator Error Prediction

This section describes an error prediction procedure to determine which structure–substructure combinations result in increased identification error. Previous work by Zhang and Johnson (2012a) show that decreased interstory acceleration response in a particular frequency range predicts which stories result in increased identification error. Therefore, the procedure is based on a weighted integral of the interstory response.

This study argues that the identification error of a substructure estimator is related to the weighted integral of the numerator of H_{EST} where the weight function is H_{MOD} . For the shear building estimator, the performance prediction function P_i is

$$P_{i} = \int_{\omega_{l}}^{\omega_{u}} \left| W(j\omega) \left[H_{\ddot{x}_{i},\ddot{u}_{g}}(j\omega) - H_{\ddot{x}_{i+1},\ddot{u}_{g}}(j\omega) \right] \right| d\omega$$
(4.14)


Figure 4.4: Bar plot of various prediction functions for the testbed structure normalized by the weight function.

where $W(j\omega) = H_{\text{MOD}}(j\omega)$; the limits of integration ω_u and ω_l are the end points of the identification bandwidth as described in Section 3.3; and *i* is the story to be identified.

Zhang and Johnson (2012a) use a different performance function to characterize identification performance. The weighting function is a notch band-pass filter that selects the substructure natural frequency $\omega_0 = \sqrt{k_i/m_i}$. Likewise, a unity weighting function can be used instead of either of the two weighting functions.

The performance predictions are compared for each of these weighting functions in Figure 4.4. It is clear that all three different weighting functions make the same predictions. Specifically, the eighth story is predicted to have the greatest identification error because it has the lowest interstory acceleration response.

Chapter 5

Nonlinear Regression

The model function of the substructure estimator is a nonlinear function of the parameters and is equivalent to the transfer function of a SDOF oscillator. To estimate the parameters, nonlinear regression is performed. By assuming a standard regression structure, the problem can be written as

$$y_n = \eta(\omega_n, \boldsymbol{\theta}) + \varepsilon_n \tag{5.1}$$

where the equation is indexed on n = 1, 2, ..., N, the set of observations (the statistical degrees of freedom). The error, ε_n , is a random variable that is a combination of the measurement noise, FRF estimation error, and nonlinear function estimation error. It is assumed that ε_n is adequately represented by a Gaussian random variable with equivalent first and second statistical moments. θ is a $N_{\theta} \times 1$ vector containing the parameter values to be identified, ω_n is the frequency of observation n, $\eta(\omega_n, \theta) = H_{\text{MOD}}(j\omega_n)$ is the model function, and $y_n = H_{\text{EST}}(j\omega_n)$ is the corresponding estimate of the nonlinear function of estimated quantities.

By casting the regression problem in a general nonlinear regression form such as (5.1), a wide body of statistical literature can be leveraged to find an efficient solution. This chapter,

first describes the optimal estimate using least squares (LS) and maximum likelihood (ML) estimation. Second, confidence regions are constructed using several different methods. Third, statistical curvature is presented as a prediction of hypothesis testing performance. Fourth, a LS error analysis is presented and compared with a previous error analysis.

5.1 Estimation

This section develops two techniques for optimal estimation, LS and ML estimation. LS estimation finds an optimal estimate by minimizing the sum of the squares of residuals. ML estimation finds the optimal estimate that maximizes the likelihood function. The identification performance of these two methods is compared.

5.1.1 Least Squares Estimation

The least squares estimate (LSE) is given by minimizing the sum of the squares of residuals. By assuming the errors are zero-mean, independent and identically distributed (i.i.d.) as circularly symmetric complex Gaussian random variables, the LSE is a maximum likelihood estimate (MLE) (Mendel, 1995). This assumption allows the complex valued functions to be represented succinctly by the squared norm of the residual because the real and imaginary components are independent and, thus, decomposed.

$$S(\boldsymbol{\theta}) = \sum_{n=1}^{N} |\eta(\omega_n, \boldsymbol{\theta}) - y_n|^2$$
(5.2)

where the LSE is the value of θ that minimizes $S(\theta)$.



Figure 5.1: Least square estimator surface for third-story parameter estimation in the testbed structure.

There are many benefits of using LS. First, the particular distribution of the error is not needed. Therefore, only the estimate of the nonlinear function is needed and as such, the optimal estimate can be used $y_n = H_{\text{EST}}(\mu_z)$. Second, the LSE is unbiased and efficient. Third, assuming a small set of parameter values close to the actual values, LS optimization is convex. The estimator surface is shown in Figure 5.1 for the testbed problem (*i.e.*, estimating third-story parameters in the testbed structure).

LS has two major drawbacks. First, the assumption that errors ε_n are circular is false because, at various frequency values, the correlation between real and imaginary components is non-zero. Second, the assumption that errors are identically distributed is

false because larger error variance is found at frequency values with comparatively higher system response.

Another concern is the heteroskedasticity encountered during substructure identification. This behavior is the result of the model function having higher system response near the substructure natural frequency. This behavior is unavoidable and its implications will be further discussed in Section 6.4.1.

5.1.2 Maximum Likelihood Estimation

The maximum likelihood estimate (MLE) is found by maximizing the log-likelihood function. This requires knowledge of the underlying probability distribution. It is assumed herein, that the error is distributed as a complex Gaussian random variable or can be approximated by the second-moment statistics equivalent to a complex Gaussian random variable. This assumption is distinct from the assumptions required for LS because the circular symmetry and identical distribution requirements are released. It is still assumed that errors are independent from one frequency point to another, which is reasonable and consistent with standard Fourier transform theory. Using these assumptions, the MLE is found by minimizing the log-likelihood function.

$$L(\boldsymbol{\theta}) = \sum_{n=1}^{N} \left[\left(\frac{\Re\{\eta(\omega_n, \boldsymbol{\theta}) - y_n\}}{\sigma_n^{\Re}} \right)^2 - 2\rho_n \left(\frac{\Re\{\eta(\omega_n, \boldsymbol{\theta}) - y_n\}}{\sigma_n^{\Re}} \right) \left(\frac{\Im\{\eta(\omega_n, \boldsymbol{\theta}) - y_n\}}{\sigma_n^{\Im}} \right) + \left(\frac{\Im\{\eta(\omega_n, \boldsymbol{\theta}) - y_n\}}{\sigma_n^{\Im}} \right)^2 \right]$$

$$(5.3)$$



Figure 5.2: Maximum likelihood estimator surface for third-story parameter estimation in the testbed structure.

Here σ_n^{\Re} and σ_n^{\Im} are the standard deviations of the error, at a particular frequency value ω_n in the real and imaginary directions, respectively. Likewise, ρ_n is the correlation coefficient between the real and imaginary components of the error at ω_n . $\eta(\omega_n, \theta)$ and y_n are, as previously defined.

The major benefit of the MLE over LSE is that it removes much of the observed heteroskedasticity by de-weighting at frequency values of high system response that are found to have proportionally higher error variance. Additionally, ML estimation is theoretically more sound in that correlation effects between the real and imaginary components are included. However, this was found to have minimal effect on the performance of the estimators.

The major drawback is that the MLE requires second-moment statistics of the error distribution, which cannot be computed directly using the optimal estimate of the nonlinear function. Therefore, to use ML, it is necessary to use the sub-optimal estimate of the nonlinear function or use *a priori* knowledge of error distribution.

5.1.3 Comparison

The two estimates are compared using the testbed structure. The the mean and standard deviation of the identified stiffness is shown in Table 5.1. The MLE demonstrates slightly decreased error variance and modest bias error in most stories. Both estimates admit suitable performance.

While the LSE is susceptible to heteroskedasticity, the MLE is largely resistant because the locations of high system response carry similarly high levels of variance, which are used to de-weight that sample. This property is a suitable explanation for the decreased error variance observed in the identification results.

5.2 Confidence Regions

The cornerstone of successful damage detection is uncertainty quantification. The most user friendly description is through confidence regions that allow the user to specify a desired level of confidence and determine if damage has occurred. A confidence region is the set of parameter values that can be expected to contain the real parameter value to a

	LS	E	MLE			
	Mean	Std	Mean	Std		
Story	[%]	[%]	[%]	[%]		
1	-1.15	0.07	-1.36	0.06		
2	-0.01	0.11	-0.02	0.08		
3	-0.00	0.12	-0.03	0.11		
4	-0.02	0.17	-0.05	0.14		
5	-0.11	0.28	-0.08	0.19		
6	-0.05	0.22	-0.03	0.19		
7	-0.01	0.29	0.10	0.25		
8	-0.41	0.80	-0.40	0.49		
9	-0.16	0.32	-0.05	0.26		
10	0.08	0.20	0.03	0.17		

Table 5.1: Substructure identification statistics of testbed structure for LS and ML estimates.

specific probability conditioned on the data observed. The confidence region is a function of the data defined $CR_{\alpha} := \mathbb{R}^N \mapsto \mathbb{R}^{N_{\theta}}$ such that¹

$$\Pr\left\{\boldsymbol{\theta} \in CR_{\alpha}(\mathbf{y})\right\} = 1 - \alpha \tag{5.4}$$

where θ is the true parameter value; **y** is the observed data vector as defined in (5.1); and α is the specified level of confidence.

A N_{θ} -dimensional confidence region can be projected onto a particular parameter's axis to generate a confidence interval. The interval (or set of intervals) is a function defined $CI_{l,\alpha} := \mathbb{R}^N \mapsto \mathbb{R}$ such that,

$$\Pr\left\{\theta_l \in CI_{l,\alpha}(\mathbf{y})\right\} = 1 - \alpha \tag{5.5}$$

¹Here the confidence region is defined as real function while the model function and data vector are complex-valued. Under the assumption that the real and imaginary components are independent, the complex-valued vector \mathbb{C}^N is decomposed into a real-valued vector \mathbb{R}^{2N} . This assumption is met in this study.

where θ_l is the entry of θ that is being tested.

Once the confidence region for a particular realization is constructed, the analyst can perform damage detection using substructure identification. If the nominal value of the stiffness of a particular story is contained within the computed confidence region, it is reasonable to conclude that the structure is undamaged at that story. This is implemented within a hypothesis testing framework where the null hypothesis is taken as the undamaged case.

The damage detection performance of a hypothesis test can be evaluated through MCS. For each realization, the confidence region is computed and it is recorded whether the nominal parameter is contained in the region. The statistical coverage is defined as the percentage of realizations in which the null hypothesis (undamaged case) is returned. For a properly constructed confidence region and an undamaged structure, it is expected that the coverage should be $100(1 - \alpha)\%$.

This section will describe two methods of creating confidence regions. The first confidence region uses a linear approximation based on the LSE to construct elliptic confidence regions. The second confidence region uses the likelihood function (5.3) to construct the ML confidence region. The section concludes with a direct comparison of the two methods evaluated by their observed coverage.

5.2.1 Linear Confidence Region

A linear confidence region is constructed using a linear approximation of the nonlinear model function. This approximation is equivalent to assuming that the solution locus is planar and that the coordinate grid is linear throughout the confidence region (Donaldson and Schnabel, 1987). Under these assumptions, the confidence region is an ellipse that is defined,

$$CR_{\alpha} = \left\{ \boldsymbol{\theta} := \left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\right)^{\mathsf{T}} \hat{\mathbf{V}}^{-1} (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \le N_{\theta} F_{N_{\theta}, N - N_{\theta}}^{1 - \alpha/2} \right\}$$
(5.6)

where $\hat{\theta}$ is the LSE; $\hat{\mathbf{V}}$ is the estimated error covariance matrix; and $F_{N_{\theta}, N-N_{\theta}}^{1-\alpha/2}$ is a specific value of the Fisher distribution.

The only value in (5.6) that is not previously given is the estimated error covariance matrix. This quantity can be computed using the linear approximation in two ways. The first method of estimating **V** computes the error covariance matrix by using the Jacobian of the model function; this is equivalent to using the expected Fisher Information matrix \mathbb{E} [**FIM**]. The second method uses the Hessian of the estimation functional ($S(\theta)$ for LSE and $L(\theta)$ for MLE) with respect to the identified parameters and is equivalent to using the observed Fisher Information matrix **FIM** (Donaldson and Schnabel, 1987). The equations are shown below:

$$\hat{\mathbf{V}}_{\mathbf{e}} = \hat{\sigma}^2 \left(\mathbf{J}_{\text{MOD}}(\hat{\boldsymbol{\theta}}) \mathbf{J}_{\text{MOD}}^{\mathsf{H}}(\hat{\boldsymbol{\theta}}) \right)^{-1} = \hat{\sigma}^2 \left(\mathbb{E} \left[\mathbf{FIM} \right] \right)^{-1}$$
(5.7)

$$\hat{\mathbf{V}}_{\mathrm{o}} = \hat{\sigma}^2 \mathbf{H}(\hat{\boldsymbol{\theta}})^{-1} = \hat{\sigma}^2 \left(\widehat{\mathbf{FIM}}\right)^{-1}$$
(5.8)

where $\mathbf{J}_{\text{MOD}}^{\text{H}}(\hat{\boldsymbol{\theta}})$ is the Jacobian of the model function $\boldsymbol{\eta}(\boldsymbol{\theta})$ evaluated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$ (*i.e.*, $[J_{\text{MOD}}^*]_{(l,n)} = \partial H_{\text{MOD}}(j\omega_n)/\partial \theta_l|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}}$); $\mathbf{H}(\hat{\boldsymbol{\theta}})$ is the Hessian of $S(\hat{\boldsymbol{\theta}})$ or $L(\boldsymbol{\theta})$ evaluated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$; and $\hat{\sigma}^2$ is the residual variance $\hat{\sigma}^2 = S(\hat{\boldsymbol{\theta}})/(N - N_{\theta})$.

Using the estimated error variance $\hat{\mathbf{V}}$, a hypothesis test is constructed as,

$$\mathcal{H}_{0} \colon (\boldsymbol{\theta} - \boldsymbol{\hat{\theta}})^{\mathsf{T}} \hat{\mathbf{V}}^{-1} (\boldsymbol{\theta} - \boldsymbol{\hat{\theta}}) \leq N_{\theta} F_{N_{\theta}, N - N_{\theta}}^{1 - \alpha/2} \quad \text{(undamaged)}$$

$$\mathcal{H}_{1} \colon (\boldsymbol{\theta} - \boldsymbol{\hat{\theta}})^{\mathsf{T}} \hat{\mathbf{V}}^{-1} (\boldsymbol{\theta} - \boldsymbol{\hat{\theta}}) > N_{\theta} F_{N_{\theta}, N - N_{\theta}}^{1 - \alpha/2} \quad \text{(damaged)}$$
(5.9)

where θ is the true parameter value and \mathcal{H}_0 is the null hypothesis that the identified parameter θ indicates that the identified story is undamaged.

Since the linear confidence region is an ellipse, it is easy to construct a single confidence interval for a given parameter entry. This confidence interval, based on Student's T distribution, assumes that the parameter is estimated from a series of normally distributed data points. This value is multiplied by the standard deviation $\hat{V}_{l,l}$ of the estimated parameter $\hat{\theta}_l$. The confidence interval is the set of points satisfying:

$$CI_{l,\alpha} = \left\{ \theta_l := \left| \theta_l - \hat{\theta}_l \right| \le \sqrt{\hat{V}_{l,l}} t_N^{1-\alpha/2} \right\}$$
(5.10)

where $\hat{\theta}_l$ is the identified parameter; $t_N^{1-\alpha/2}$ is the realization of Student's T distribution; and $\hat{V}_{l,l}$ is the estimated error variance of the identified parameter.

As written, (5.10) provides a confidence interval for one of the identified parameters, which can be repeated for every identified parameter of interest. This can be used in a hypothesis testing framework to determine if a specific parameter is damaged. The hypothesis test is defined,

$$\mathcal{H}_{0} \colon \left| \theta_{l} - \hat{\theta}_{l} \right| \leq \sqrt{\hat{V}_{l,l}} t_{N}^{1-\alpha/2} \quad \text{(undamaged)}$$

$$\mathcal{H}_{1} \colon \left| \theta_{l} - \hat{\theta}_{l} \right| > \sqrt{\hat{V}_{l,l}} t_{N}^{1-\alpha/2} \quad \text{(damaged)}$$
(5.11)

where \mathcal{H}_0 is the null hypothesis that the identified parameter θ_l is undamaged.

In this section, the vector-based hypothesis test given by (5.9) will be used because it offers a more direct comparison to the ML confidence region. However, in later chapters, the hypothesis test given by (5.11) will be used to determine if the stiffness has changed. This choice allows for a more direct detection of stiffness changes that forms the basis of the damage detection framework.

This subsection concludes by noting that two different methods of estimating the error covariance matrix are developed. These two methods use the same assumptions and data to construct different estimates. When the model function is linear these two methods simplify to the same linear solution. Moreover, Marsili-Libelli et al. (2003) used these two methods as an outlier rejection mechanism when the two predicted error variances from (5.7) and (5.8) were not similar. They found that they were able to reject estimates where the deterministic optimizer did not converge to the correct value.

5.2.2 Maximum Likelihood Confidence Regions

The ML method sidesteps any direct estimate of variance and instead relies on a properly specified log-likelihood function (5.3). Using this equation, contours of equal likelihood can be constructed for various realizations of θ . Moreover, the various contours have additional meaning because the likelihood function is stochastically weighted to reflect the overall variance of the estimate. Therefore, the confidence region can be computed to a specific confidence level by comparing the likelihood ratio to the F-distribution

	LS	SE	MLE		
Story	Ve	Vo	Vo	ML	
1	0.00	0.00	0.00	0.00	
2	91.73	71.38	81.97	81.72	
3	94.02	75.75	78.67	78.20	
4	94.16	76.19	80.25	79.71	
5	89.83	68.41	75.12	74.63	
6	92.77	73.21	51.19	50.21	
7	94.09	76.06	66.94	65.91	
8	89.15	67.41	28.92	27.61	
9	92.45	72.60	47.10	45.83	
10	88.51	65.47	87.73	87.67	

Table 5.2: Statistical coverage of identified story stiffness for various types of confidence intervals with a 95% confidence level. Statistical coverage is the percentage of undamaged hypothesis results.

(Donaldson and Schnabel, 1987). The α confidence region for the identified parameter vector $\hat{\theta}$ is defined as

$$CR_{\alpha} = \left\{ \boldsymbol{\theta} : L(\boldsymbol{\hat{\theta}}) - L(\boldsymbol{\theta}) < \hat{\sigma}^2 F_{N_{\theta}, N-N_{\theta}}^{1-\alpha/2} \right\}$$
(5.12)

This confidence region is non-elliptic and can be visualized as the level curve along the likelihood surface. Therefore, it is computationally expensive to compute a particular confidence region using the ML method. Fortunately, (5.12) can be used in a hypothesis testing framework, which requires only two function evaluations of (5.3) and is very computationally efficient. The hypothesis test is

$$\mathcal{H}_{0}: L(\hat{\boldsymbol{\theta}}) - L(\boldsymbol{\theta}) \leq \hat{\sigma}^{2} F_{N_{\theta}, N-N_{\theta}}^{1-\alpha/2} \quad \text{(undamaged)}$$

$$\mathcal{H}_{1}: L(\hat{\boldsymbol{\theta}}) - L(\boldsymbol{\theta}) > \hat{\sigma}^{2} F_{N_{\theta}, N-N_{\theta}}^{1-\alpha/2} \quad \text{(damaged)}$$
(5.13)

where \mathcal{H}_0 is the null hypothesis that the identified parameter vector $\boldsymbol{\theta}$ is undamaged.

5.2.3 Comparison

The linear confidence region defined in (5.6) is compared to the ML confidence region defined in (5.12). The linear confidence region is generated using two different estimates of the error covariance matrix and the LSE. Likewise, the MLE is used for a linear confidence region and the ML confidence region. The statistical coverage is computed for these different combinations and are compared in Table 5.2.

The results show that the linear confidence interval computed using the expected Fisher Information matrix ("LSE V_e ") outperforms the other methods with observed coverage approaching the nominal value, 95%, at most stories. In all of the methods, the first story is detected as damaged as a result of the bias error for each estimate (see Table 5.1). This could be overcome in practice by using the biased value of the stiffness but is not performed in this study for consistency.

5.3 Statistical Curvature

Whenever a nonlinear model function is used in regression, it is important to check the validity of linearizing assumptions. While nonlinear estimators can be formulated without linear assumptions, much of their performance and confidence region computations rely on a local linear model. Therefore, it is important to characterize the levels of non-linearity.

The first work to characterize the amount of non-linearity in a model equation was performed by Beale (1960), who developed a measure of non-linearity that could be used to reject certain model equations. Later, Bates and Watts (1980) developed two measures of non-linearity that could be related to Beale's measure but were rigorously derived using differential geometry. These two measures determine the magnitude of curvature in the solution locus and predict problems with a linear assumption.

The first curvature measure is the relative intrinsic curvature that represents deviation from the planar assumption in the solution locus. The second measure is the relative parameter-effects curvature that represents a deviation from the uniform coordinate assumption. These two measures are computed by noting that the model function maps the *N*-dimensional space to an N_{θ} -dimensional subspace. When the surface is not locally planar at the estimated point, intrinsic curvature is increased. Likewise, when the coordinate grid that locates different parameter values deviates from an orthogonal coordinate system, parameter-effects curvature is increased. Note, the uniform coordinate assumption can be violated by expansion/compression, arcing, and/or fanning of the coordinate grid (Bates and Watts, 2007).

In the specific model described herein, relative intrinsic curvature and parametereffects curvature were observed to be small. This indicates that confidence intervals can be created using a linear approximation.

5.3.1 Derivation

The derivation of the maximum intrinsic curvature Γ^{N} and the maximum parameter effects curvature² Γ^{T} follows from Bates and Watts (1980).

First, using the notation of the LS regression, write the sum of the squares as the norm of the error.

$$S(\boldsymbol{\theta}) = \sum_{n=1}^{N} |\eta(\omega_n, \boldsymbol{\theta}) - y_n|^2 = \|\mathbf{y} - \boldsymbol{\eta}(\boldsymbol{\theta})\|_2^2$$
(5.14)

²The maximum curvature Γ^{T} is a scalar value and should not be confused with the transpose operation $(\cdot)^{T}$.

where $\eta_n(\theta) = \eta(\omega_n, \theta)$. The solution locus $\eta(\theta)$ is a N_{θ} -dimensional surface projected onto the *N* dimensional sample space where N_{θ} is the number of parameters to be estimated (*i.e.*, $\eta(\theta)$ is $\mathbb{R}^{N_{\theta}} \to \mathbb{R}^{N}$). In general, **y** is not a point on the solution locus and the LSE $\hat{\theta}$ is the parameter value where the solution locus is closest to **y**.

The uncertainty of a particular estimate is described by the behavior of the solution locus in the neighborhood of the estimate. The most common description in statistical literature is linear confidence regions and intervals. To compute these, a linear approximation of the solution locus is used, which is equivalent to a tangent plane representation evaluated at the estimate $\hat{\theta}$. The geometric effect of using this approximation is to assume that the solution locus is planar and the coordinates are uniform.

The two curvature measures developed by Bates and Watts (1980) evaluate the deviations from the planar and uniform coordinates assumptions. They do this by measuring the curvature of the solution locus, which is simply the "acceleration" of the surface divided by the "velocity" of the surface properly normalized. Specifically, the velocity and acceleration can be computed in terms of a *lifted line*.

An arbitrary straight line in the parameter space, through a particular point θ_0 , can be expressed using a geometric parameter *b* as

$$\boldsymbol{\theta}(b) = \boldsymbol{\theta}_0 + b\mathbf{h} \tag{5.15}$$

where **h** is any non-zero vector of dimension N_{θ} . This generates a lifted line on the solution locus, $\eta_{\mathbf{h}}(b)$

$$\eta_{\mathbf{h}}(b) = \eta(\theta_0 + b\mathbf{h}) \tag{5.16}$$

75

Using the lifted line representation in (5.16), the velocity and acceleration of the solution locus can be computed in direction **h** and at a particular point θ_0 (*i.e.*, at b = 0).

$$\dot{\boldsymbol{\eta}}_{\mathbf{h}} = \frac{d\boldsymbol{\eta}_{\mathbf{h}}}{db}\Big|_{b=0} = \mathbf{V} \mathbf{h}$$
(5.17)

$$\ddot{\boldsymbol{\eta}}_{\mathbf{h}} = \frac{d^2 \boldsymbol{\eta}_{\mathbf{h}}}{db^2} \Big|_{b=0} \quad \text{where} \quad \ddot{\boldsymbol{\eta}}_{\mathbf{h},n} = \frac{d^2 \boldsymbol{\eta}_{\mathbf{h},n}}{db^2} \Big|_{b=0} = \mathbf{h}^{\mathsf{T}} \{ \mathbf{V}_{..} \}_n \mathbf{h}$$
(5.18)

where \mathbf{V}_{\cdot} is an $N \times N_{\theta}$ matrix whose i^{th} column is $\partial \eta / \partial \theta_i |_{\theta = \theta_0}$ (*i.e.*, the Jacobaian of $\eta(\theta)$) and \mathbf{V}_{\cdot} is an $N_{\theta} \times N_{\theta}$ array of N dimensional vectors where $\{\mathbf{V}_{\cdot}\}_n$ is the Hessian of $\eta_n(\theta)$.³

Additionally, the acceleration can be decomposed into three components with geometric significance.

$$\ddot{\boldsymbol{\eta}}_{\mathbf{h}} = \ddot{\boldsymbol{\eta}}_{\mathbf{h}}^{\mathrm{N}} + \ddot{\boldsymbol{\eta}}_{\mathbf{h}}^{\mathrm{G}} + \ddot{\boldsymbol{\eta}}_{\mathbf{h}}^{\mathrm{P}} \tag{5.19}$$

where $\ddot{\eta}_{h}^{N}$ is the change of direction of $\dot{\eta}_{h}$ normal to the tangent plane; $\ddot{\eta}_{h}^{G}$ is the change of direction of $\dot{\eta}_{h}$ parallel to the tangent plane; and $\ddot{\eta}_{h}^{P}$ is the change of speed of the moving point and hence determines whether the point moves uniformly across the solution locus (Bates and Watts, 1980).

Using this decomposition of the acceleration, the curvature in a particular direction can be defined. As such, the normal curvature $K_{\mathbf{h}}^{N}$ in direction **h** is

$$K_{\mathbf{h}}^{\mathbf{N}} = \frac{\|\ddot{\boldsymbol{\eta}}_{\mathbf{h}}^{\mathbf{N}}\|}{\|\dot{\boldsymbol{\eta}}_{\mathbf{h}}\|^2} \tag{5.20}$$

 $^{{}^{3}}V_{.}$ and $V_{..}$ are geometric derivative-like terms that are used to determine the curvature. Subscripted dots representing the derivative are used to not confuse with the standard over-dot notation of time derivatives.

and the parameter effects curvature $K_{\mathbf{h}}^{\mathrm{T}}$ in direction **h** is

$$K_{\mathbf{h}}^{\mathrm{T}} = \frac{\|\ddot{\boldsymbol{\eta}}_{\mathbf{h}}^{\mathrm{G}} + \ddot{\boldsymbol{\eta}}_{\mathbf{h}}^{\mathrm{P}}\|}{\|\dot{\boldsymbol{\eta}}_{\mathbf{h}}\|^{2}}$$
(5.21)

If the curvatures $K_{\mathbf{h}}^{\mathbf{N}}$ and $K_{\mathbf{h}}^{\mathbf{T}}$ are large relative to the curvature of the confidence region, $1/\sqrt{\rho F(N_{\theta}, N, \alpha)}$, then the confidence region is not adequately representing the uncertainty of the identified parameters. Moreover, the curvature is strictly non-negative and is only zero when the model function is linear in the parameters. Bates and Watts (1980) propose that models with maximum curvature less than half of the confidence region curvature should reasonably admit the linear approximation.

As derived, the curvatures, K_h^N and K_h^T , only compute the curvature in a particular direction **h**. Thus, an exhaustive search of the parameter space is required to fully understand deviations from the linear approximation. To overcome this limitation, Bates and Watts (1980) developed a procedure to calculate the maximum intrinsic curvature and maximum parameter-effects curvature, Γ^N and Γ^T , respectively. Bates and Watts find that this formulation is most effective at signaling potential problems with a particular model function. The implementation follows from the derivation and utilizes a parameter transformation involving the QR factorization of **V** along with a recursive procedure. Interested readers are referred to Bates and Watts (1980, 2007).

5.3.2 Shear Building Example

The curvature methods derived in the previous section can be computed for a particular model function. In the context of substructure identification, this applies to H_{MOD} . By

computing the curvature measures and comparing them to the curvature of the confidence region, the analyst can determine whether linear confidence regions are appropriate.

In this section, the curvature measures will be computed, as described previously, for the shear building. In order to compute the curvature measures, the complex-valued function needs to be represented as two independent real-valued functions. This representation is consistent with the assumptions made for LS estimation. Thus, the parameter vector is $\boldsymbol{\theta} = [k_i, c_i]^{\mathsf{T}}$ and the model function is

$$\boldsymbol{\eta}(\boldsymbol{\theta}) = \left[\Re\left\{\eta(\omega_1, \boldsymbol{\theta})\right\}, \Im\left\{\eta(\omega_1, \boldsymbol{\theta})\right\}, \dots, \Re\left\{\eta(\omega_N, \boldsymbol{\theta})\right\}, \Im\left\{\eta(\omega_N, \boldsymbol{\theta})\right\}\right]^{\mathsf{T}}$$
(5.22)

where $\eta(\omega_n, \theta) = H_{\text{MOD}}(j\omega_n)|_{\theta}$ so that the model function $\eta(\theta)$ contains, interleaved, the real and imaginary components of $H_{\text{MOD}}(j\omega_n)$.

Using this structure, the curvature measures are computed for different levels of frequency domain discretization. This discretization is controlled by the number of FFT points used N_{FFT} . In addition to changing the discretization, the model function can be re-parameterized. For comparison, two more parameterizations are considered in addition to H_{MOD} given in (4.1). The three parameterizations considered are as follows,

$$H_{\text{MOD}}: [\bar{k}_i, \bar{c}_i]^{\mathsf{T}} \qquad \mapsto \quad \frac{1}{1 + \bar{c}_i/s + \bar{k}_i/s^2}$$
(5.23a)

$$H_{\text{MOD}}: [\omega_0, \zeta_0]^{\mathsf{T}} \longrightarrow \frac{1}{1 + 2\zeta_0 \omega_0 / s + \omega_0^2 / s^2}$$
 (5.23b)

$$H_{\text{MOD}}: [\bar{k}_i, \bar{c}_i, \bar{m}_i]^{\mathsf{T}} \mapsto \frac{1}{\bar{m}_i + \bar{c}_i/s + \bar{k}_i/s^2}$$
(5.23c)

Table 5.3: Statistical curvature measures and coverage for various model functions and coverage for the third story of the testbed structure.

θ	N _{FFT}	Γ^{N}	Γ^{T}	F–coverage	t-coverage	
$[k_i, c_i]^T$	2 ⁸	0.4726	0.3846	0.9909	0.8630	
$[k_i, c_i]^T$	2 ¹²	0.0857	0.0683	0.9280	0.9043	
$\left[\omega_0,\zeta_0\right]^T$	2 ¹²	0.0879	0.0683	0.9322	0.9080	
$\left[k_i, c_i, m_i\right]^T$	2 ¹²	0.1029	0.0883	0.9469	0.9571	

where k_i , \bar{c}_i , \bar{m}_i are the mass-normalized stiffness, damping, and mass story parameters, respectively. ω_0 and ζ_0 are the substructure natural frequency and damping ratio, respectively. Additionally, (5.23c) over-parameterizes H_{MOD} by introducing \bar{m}_i . This parameterization is presented for academic purposes and does not represent a true identification of the system.

The computed curvature measures are reported in Table 5.3. It is clear that as N_{FFT} is increased, the maximum curvature decreases. Moreover, the curvature measure is much smaller than the confidence region curvature for the standard simulation properties $(N_{\text{FFT}} = 2^{12})$ and acceptable for roughly 2^{10} or larger $(N_{\text{FFT}} = 2^8 \text{ is right at the border})$. This ensures that the linear confidence regions are suitable for H_{MOD} . As discussed above, a maximum curvature less than half the confidence region is considered by Bates and Watts (1980) to be acceptable. For this choice of model function, H_{MOD} , this condition is met for all levels of discretization considered. This is important as complete time histories are often not available and coarser discretizations must sometimes be used.

5.4 Error Analysis

Using the LSE developed in Section 5.1, a linear error analysis can be performed. This analysis provides a performance measure of a particular structure–substructure estimator and is used to motivate controlled substructure identification and develop optimal sensor placements.

The first step in developing an analytical representation for substructure error analysis is to find the error for a general LS problem. This will be discussed in Section 5.4.1. Following this, the error analysis will be applied to the shear building example and results will be summarized (Section 5.4.2).

5.4.1 Least Square Error Analysis

The error analysis will be developed for a LSE by computing the expected value of the sum of the squares under FRF estimation uncertainty. Start by simplifying the notation of (5.2) and using vector notation.

$$S(\boldsymbol{\theta}) = \sum_{n=1}^{N} |H_{\text{MOD}}(j\omega_n, \boldsymbol{\theta}) - H_{\text{EST}}(j\omega_n)|^2$$

= $||\boldsymbol{\eta} - \mathbf{y}||_2^2$
= $(\boldsymbol{\eta} - \mathbf{y})^{\text{H}}(\boldsymbol{\eta} - \mathbf{y})$
= $\boldsymbol{\eta}^{\text{H}}\boldsymbol{\eta} - \boldsymbol{\eta}^{\text{H}}\mathbf{y} - \mathbf{y}^{\text{H}}\boldsymbol{\eta} + \mathbf{y}^{\text{H}}\mathbf{y}$ (5.24)

where $\eta_n = H_{\text{MOD}}(j\omega_n, \theta)$ and $y_n = H_{\text{EST}}(j\omega_n)$. Next, take the expected value of (5.24) and decompose the random vector of estimated quantities into the sum of its mean vector μ_y with a zero-mean random variable ε .

$$\mathbb{E}\left[S(\theta)\right] = \mathbb{E}\left[\eta^{\mathsf{H}}\eta - \eta^{\mathsf{H}}\mathbf{y} - \mathbf{y}^{\mathsf{H}}\eta + \mathbf{y}^{\mathsf{H}}\mathbf{y}\right]$$

$$= \mathbb{E}\left[\eta^{\mathsf{H}}\eta - \eta^{\mathsf{H}}(\mu_{\mathbf{y}} + \varepsilon) - (\mu_{\mathbf{y}} + \varepsilon)^{\mathsf{H}}\eta + (\mu_{\mathbf{y}} + \varepsilon)^{\mathsf{H}}(\mu_{\mathbf{y}} + \varepsilon)\right]$$

$$= \eta^{\mathsf{H}}\eta - \eta^{\mathsf{H}}\mu_{\mathbf{y}} - \mu_{\mathbf{y}}^{\mathsf{H}}\eta + \mu_{\mathbf{y}}^{\mathsf{H}}\mu_{\mathbf{y}} + \mathbb{E}\left[\varepsilon^{\mathsf{H}}\varepsilon\right]$$

$$= (\eta - \mu_{\mathbf{y}})^{\mathsf{H}}(\eta - \mu_{\mathbf{y}}) + \mathbb{E}\left[\varepsilon^{\mathsf{H}}\varepsilon\right]$$
(5.25)

When the model function parameters θ are exactly identified, $\eta - \mu_y = 0$. Moreover, the remaining expectation term is simply the trace of the covariance matrix \mathbf{K}_y of the function of estimated quantities. Thus, (5.25) becomes

$$\mathbb{E}\left[S(\tilde{\boldsymbol{\theta}})\right] = \mathbb{E}\left[\boldsymbol{\varepsilon}^{\mathsf{H}}\boldsymbol{\varepsilon}\right] = tr\left\{\mathbf{K}_{\mathsf{y}}\right\}$$
(5.26)

where $\tilde{\theta}$ is the true value of θ . This assumption is appropriate when the identified parameters are close to the true value and results in $\eta - \mu_y = 0$. If this condition is not met, there will be an additional bias term. In this section, only uncertainty contributions from estimated quantities are considered and the bias effects of mis-identified parameters will be ignored.

Using the results of Section 4.2 and (4.8), the expected value of the sum of the squares is approximately the sum of the variance of the function of estimated quantities at each frequency value using a linear approximation for the function.

$$\mathbb{E}\left[S(\tilde{\boldsymbol{\theta}})\right] = tr\left\{\mathbf{K}_{\mathbf{y}}\right\} \approx \sum_{n=1}^{N} \left(\mathbf{h}_{\text{EST}}^{(n)}\right)^{\mathsf{H}} \mathbf{K}_{\mathbf{z}}^{(n)} \mathbf{h}_{\text{EST}}^{(n)}$$
(5.27)

where $(\mathbf{h}_{\text{EST}}^{(n)})^{\text{H}}$ is the Jacobian of $H_{\text{EST}}(j\omega_n; \mathbf{z})$ with respect to the input random vector \mathbf{z} . Likewise, $\mathbf{K}_{\mathbf{z}}^{(n)}$ is the covariance matrix of the input random vector at the frequency ω_n . Input random vector \mathbf{z} is a vector that contains the estimated FRFs and *a priori* parameters for a particular substructure. In the common example of a chain structure, this would be the FRF of the floor level below, at, and above the story being identified along with the stiffness and damping parameters of the story above.

By assuming that the input random vector is comprised of independent circular complex Gaussian random variables, $\mathbf{K}_{\mathbf{z}}^{(n)}$ becomes a diagonal matrix and (5.27) can be further simplified

$$\mathbb{E}\left[S(\tilde{\boldsymbol{\theta}})\right] \approx \sum_{n=1}^{N} \sum_{m=1}^{M} |\{h_{\text{EST}}\}_m(j\omega_n)|^2 \sigma_{z_m}^2(j\omega_n)$$
(5.28)

where $\{h_{\text{EST}}^{(n)}\}_m$ is the m^{th} element of Jacobian $(\mathbf{h}_{\text{EST}}^{(n)})^{\mathsf{H}}$, and $\sigma_{z_m}^2(j\omega_n)$ is the (m, m) element of $\mathbf{K}_{\mathbf{z}}^{(n)}$.

The expression in (5.28) is useful for evaluating a particular realization of sensor data but is not immediately useful for control or experiment design. However, by assuming that the FRF uncertainty is a result of sensor noise with constant magnitude within the analysis bandwidth $\sigma_{z_m}^2(j\omega_n) \approx \alpha_m |H_{x_m,u}(j\omega_n)|^2$. This approximation is valid at



Figure 5.3: Jacobian of H_{EST} evaluated at various frequencies for the third story of a 10 story shear building where $\{h_{\text{EST}}\}_i = \frac{\partial H_{\text{EST}}}{\partial H_{x_i, u_g}}$

frequencies with sufficient response and deteriorates at frequencies of zero response. Violating this assumption has minimal effect on the analysis because low response implies low levels of uncertainty relative to the total analysis. This particular representation of FRF uncertainty is advocated by Pintelon and Schoukens (2001) and means that α_m can be held constant while various structural configurations or control designs can be compared.

5.4.2 Error Analysis for Shear Building

Using the result of the previous section, an approximate error analysis for a shear building structure is performed. This results in a function that predicts the level of uncertainty for a

particular substructure within a given building. This will be used to compute the relative floor identification uncertainty to predict which floors will perform worse in substructure identification.

Start by writing the error function E_i with the FRF uncertainty included as $\sigma_{z_m}^2(j\omega_n) \approx \alpha_m^2 |H_{\vec{x}_m,u}(j\omega_n)|^2$.

$$E_{i} = \mathbb{E}\left[S_{i}(\boldsymbol{\theta})\right] = \sum_{n=1}^{N} \sum_{m=i-1}^{i+1} \alpha_{m}^{2} \left|\{\mathbf{h}_{\text{EST}}\}_{m}(j\omega_{n})\right|^{2} |H_{\ddot{x}_{m},\ddot{u}_{g}}(j\omega_{n})|^{2}$$
(5.29)

where α_m is a scalar, related to the SNR, and *i* is the story-level being identified which implies that the input random vector is $\mathbf{z}_n = [H_{\ddot{x}_{i-1},\mu}(j\omega_n), H_{\ddot{x}_{i},\mu}(j\omega_n), H_{\ddot{x}_{i+1},\mu}(j\omega_n)]^T$. Thus, the expected error in identification accuracy is determined by the product of the squared magnitude of the Jacobian and each FRF of floor acceleration. Moreover, the magnitude of the same for each floor level (which is likely a result of the uniform structure assumption). Moreover, the *i*th floor acceleration is 180° out of phase with the floors below and above, as can be observed in Figure 5.3.

Figure 5.4 and Table 5.4 show the relative identification uncertainty computed using (5.29) and compared against the identification uncertainty metric used in Zhang and Johnson (2012b), defined as:

$$E_{i}^{Z} = \alpha \int_{\omega_{1}}^{\omega_{N}} \left| W(j\omega) \frac{1}{H_{(\ddot{x}_{i} - \ddot{x}_{i-1}), \ddot{u}_{g}}(j\omega)} \right|^{2} d\omega + (1 - \alpha) \int_{\omega_{1}}^{\omega_{N}} \left| W(j\omega) \frac{H_{(\ddot{x}_{i+1} - \ddot{x}_{i}), \ddot{u}_{g}}(j\omega)}{H_{(\ddot{x}_{i} - \ddot{x}_{i-1}), \ddot{u}_{g}}(j\omega)} \right|^{2} d\omega$$
(5.30)

	$\mathbb{E}\left S(\boldsymbol{\theta})_{i}\right $	E_i	E_i^Z	
Story	Observed	DeVore	Zhang	
1	0.0576	0.0234	0.1781	
2	0.0170	0.0262	0.2029	
3	0.0248	0.0230	0.1982	
4	0.0536	0.0566	0.2348	
5	0.1082	0.0844	0.3128	
6	0.0846	0.0287	0.2519	
7	0.1568	0.0978	0.2717	
8	0.9551	0.9882	0.6374	
9	0.1810	0.0321	0.2921	
10	0.0672	0.0003	0.3255	
Inner Product:		0.9819	0.8125	

Table 5.4: Substructure identification expected error predictions normalized by magnitude for both uncertainty prediction functions and the observed sum of the squares error.

where E_i^Z is the predicted error for the *i*th story; α is a scalar used to weight the effects of different types of error (in this study $\alpha = 0.8$); and the weighting function is

$$W(j\omega) = \frac{-k_i/m_i\omega^2}{1 - jc_i/m_i\omega - k_i/m_i\omega^2}$$

As the predictions do not have a physical meaning, the results are normalized so that the magnitude of the vector describing the entire structure is equal to unity. Then, the inner product is found between the predictions and the observed. The results indicate that the normalized error prediction made with (5.29) better predicts the observed error than the error prediction made with (5.30). It should be noted that the prediction is dominated by the eighth story which provides most of the magnitude.



Figure 5.4: Substructure identification expected error predictions normalized for both uncertainty prediction functions and the observed sum of the squares error.

5.5 Best Practice

In this study, best practice represents a nonlinear LS estimator (5.2) with linear confidence intervals, denoted "LSE V_e " in Table 5.2. This combination represents the best trade-off between performance and efficiency: it is both computationally efficient and out-performs other methods by achieving statistical converge closest to the value observed in MCS.

The LS estimator has greater computational efficiency than the ML estimator because it does not use estimated variance information. To compute the estimated variance, the Jacobian of H_{EST} needs to be evaluated and multiplied with the FRF covariance matrix at each frequency value. These extra function evaluations and matrix multiplications result in the LS estimator being more computationally efficient than the ML estimator.

The LS estimator has identification performance comparable to that of the ML estimator but out-performs in the computation of confidence intervals. The identification performance is very similar for the two estimators as evidenced by the results of Table 5.2. This indicates that the simplifying distribution assumptions (*i.e.*, i.i.d. circular symmetry) made for the LSE are valid and that the performance is comparable to the MLE. However, the LSE using the linear confidence interval computed with the expected Fisher Information out-performs all other estimates and is comparable to the predication using MCS variance. This conclusion is founded on the superior statistical coverage.

In addition to the LS estimator, performance can be improved through a proper selection of optimization algorithm. This study used a gradient-based optimization provided by the MATLAB command fmincon. This command uses the trust-region-reflective algorithm, which uses a gradient to iterate towards the optimal solution. This method admits satisfactory performance when appropriate bounds are selected for identified parameters $(\pm 50\%)$ and measurement noise is low (SNR > 20 dB) with good statistical properties.

When the SNR is low or the measurement noise violates Gaussian distribution requirements, the LS estimator often admits local minima and will not converge to the global optimum. In this case, a gradient-free method is used: the *Coliny Adaptive-Pattern Search* method implemented in DAKOTA (Adams et al., 2010). This method searches along a coordinate grid and avoids local minima better than other methods. However, it does suffer from dramatically increased computational cost due to the increased number of function evaluations and I/O operations between MATLAB and DAKOTA.

In conclusion, it is recommended to use a LS estimator in combination with linear confidence intervals using the expected Fisher Information which is computed from the Jacobian of the ROM. Optimization is best provided by MATLAB directly when regular conditions are met. In cases of high noise, or other issues causing local minima, DAKOTA is recommended using a gradient-free method.

Part II

Numerical Simulations

Chapter 6

Statistical Performance

This chapter will describe the simulation results for substructure identification subject to two sources of uncertainty. The first source of uncertainty considered is bias error in the previous story stiffness and the second source is additive measurement noise. Following, a damage detection scenario is simulated and substructure identification is used to detect and localize damage in the testbed structure.

To create statistically significant results, MCS is used where each scenario uses 10,000 independent simulations. Each simulation consists of a generated realization of acceleration time history that is polluted with a realization of measurement noise; data processing using the substructure estimator as discussed in Chapter 4 and nonlinear regression as described in Chapter 5.

This chapter provides the basis for evaluating the performance of a real-world implementation of substructure identification. The bias error represents any initial error in the estimate of the nominal ROM. The measurement noise can establish the trade-off

Table 6.1: Identified stiffness statistics for various levels of previous story bias. The stiffness is reported as a percentage of the actual stiffness value and each time history includes measurement noise with 30 dB SNR.

	−10% Bias		-5% Bias		0% Bias		5% Bias		10% Bias	
Story	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
1	-5.73	0.07	-3.30	0.07	-1.15	0.07	0.76	0.08	2.51	0.09
2	-5.52	0.10	-2.68	0.10	-0.01	0.11	2.36	0.12	4.31	0.14
3	-4.89	0.10	-2.56	0.11	-0.00	0.12	2.82	0.14	5.88	0.17
4	-2.75	0.15	-1.31	0.16	-0.02	0.17	1.16	0.19	2.29	0.22
5	-6.76	0.25	-3.18	0.26	-0.11	0.28	2.32	0.31	4.09	0.45
6	-6.58	0.17	-3.45	0.19	-0.04	0.22	3.67	0.26	7.71	0.33
7	-0.28	0.23	-0.13	0.26	-0.01	0.29	0.09	0.33	0.20	0.38
8	-11.45	0.87	-4.38	0.76	-0.42	0.79	1.93	2.58	-8.68	24.01
9	-8.80	0.26	-4.57	0.28	-0.16	0.32	4.50	0.38	9.47	0.46
10	0.08	0.20	0.08	0.20	0.08	0.20	0.08	0.20	0.08	0.20

between more sensitive (and more expensive) sensor systems. Finally, the damage detection scenario investigates the effect of error propagation from one story's identification to another.

6.1 Bias Error

The first source of uncertainty considered is bias error in the previous story stiffness k_{i+1} . This error enters the substructure estimator as a deterministic error in the computation of H_{EST} . Different levels of bias error are considered ranging from -10% to 10%. The results of MCS are displayed in Table 6.1 and rendered graphically in Figure 6.1.

From these results, three findings emerge. First, previous story bias error has a consistent effect on bias error in the identified stiffness parameter and generally admits smaller bias error than the input. A notable exception is the ninth story, which has roughly the same identification bias error as the input. The seventh story stiffness identification



Figure 6.1: Box plot of identified stiffness parameter for various levels of previous story bias error. The quantiles shown are 2.5%, 25%, 50%, 75%, and 97.5%.

is the least sensitive and the eighth story is the most sensitive to previous story bias error. Second, different stories are more sensitive to previous story bias error. Third, it is noted that the identification error variance is mostly resistant to previous story bias error. However, the eighth story finds a significant increase for the case of 10% bias that is the result of improper identification as H_{EST} is no longer the same form as H_{MOD} . This is seen visually in Figure 6.2, which shows the mean of H_{EST} and the standard deviation of the residual.



Figure 6.2: Plot showing the mean of H_{EST} and the standard deviation of the residual, as a function of frequency, for various levels of bias, for the eighth story.

These results should be considered valid for the case of small bias errors within the range considered. For larger bias error in previous story stiffness, identification is often unsuccessful with H_{EST} taking a form different from the model function, H_{MOD} . This behavior is seen in the edge cases (±10% bias) of the eighth story identification (see Figure 6.2). This indicates that care needs to be taken to ensure that reasonable estimates of the previous story stiffness be used.

6.2 Measurement Noise

The second source of uncertainty considered is additive measurement noise. This error enters the substructure estimator through errors in the component FRF estimates which,

	10 dB		20 dB		30 dB		40 dB	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
1	-2.10	0.94	-1.25	0.23	-1.15	0.07	-1.14	0.02
2	-1.84	1.77	-0.16	0.34	-0.01	0.11	0.01	0.03
3	-0.44	2.16	-0.03	0.38	-0.00	0.12	0.00	0.04
4	-4.66	15.65	-0.27	0.62	-0.02	0.17	-0.00	0.05
5	-20.20	6.59	-1.11	1.11	-0.11	0.27	-0.01	0.09
6	-3.83	2.65	-0.40	0.68	-0.05	0.22	-0.00	0.07
7	-27.77	41.10	-0.80	1.96	-0.01	0.29	0.00	0.09
8	-48.52	5.37	-8.45	5.17	-0.41	0.79	-0.04	0.23
9	-32.78	20.33	-1.30	1.34	-0.16	0.32	-0.02	0.10
10	0.37	6.61	0.71	0.70	0.08	0.20	0.01	0.06

Table 6.2: Identified stiffness statistics for various SNR levels. The stiffness is reported as a percentage of the actual stiffness value.

in turn, create errors in the estimation of H_{EST} . While some level of measurement noise is included in each simulation in this study (default SNR of 30 dB), varying levels of measurement noise will be simulated in this section to better understand the effects. The magnitude of measurement noise is characterized by the SNR as defined in Chapter 3.

In this study, four different SNR levels are considered: 10, 20, 30, and 40 dB. The results of the study, documented in Table 6.2 and Figure 6.3, present four general findings. First, for low levels of noise (SNR of 30 and 40 dB), the identification bias remains small. For larger levels of noise (SNR of 10 and 20 dB), significant bias is introduced. Second, the standard deviation of the identified stiffness varies logrithmically with the level of measurement noise. As the noise is increased by an order of magnitude, from SNR of 40 to 20 dB, the standard deviation of identified stiffness increases by roughly an order of magnitude in nearly every story. Third, different stories admit different performance, with the eighth story presenting the worst performance. Fourth, as the measurement noise is



Figure 6.3: Box plot of identified stiffness parameter for various levels of SNR. The quantiles shown are 2.5%, 25%, 50%, 75%, and 97.5%.

decreased, substructure identification tends to be an unbiased estimator of stiffness with the single exception of the first story.

6.3 Damage Detection

In this study, damage detection is performed using the mean stiffness and damping parameters (\bar{k}_i and \bar{c}_i) computed from Monte Carlo simulation as a baseline. These values are used to provide a more realistic representation of the intrinsic parameter uncertainty for identification and damage detection. Specifically, the MCS bias is used as the undamaged
	Table 6.3: Damage Scenarios
\mathcal{D}_0	Undamaged Structure
\mathcal{D}_1	5% Stiffness loss in Third Story
\mathcal{D}_2	5% Stiffness loss in Third Story and 5% Stiffness loss in Eighth Story

value, which can be thought of as a "burn-in" process where the baseline performance is determined.

Three different damage scenarios are investigated. The specifics of each damage scenario are shown in Table 6.3. For each trial, the top story is identified first and then, the identified stiffness value is tested using the confidence interval hypothesis test in (5.11) to predict damage. If damage is predicted, the identified value is used to compute the parameters of the story below; if no damage is predicted, the mean value for the undamaged structure is used (under the assumption that the undamaged structure has been well-characterized by many periodic tests). Then, the story beneath is identified and the process is iteratively performed until every story is identified and tested for damage. The results for each damage scenario are shown in Table 6.4 where the statistical coverage (*i.e.*, percentages of null hypothesis for each floor) is shown. Likewise, the box plot of identified stiffness is shown in Figure 6.4.

The results show that there is Type I error (false positives) and minimal Type II error (false negatives). The Type I error varies between 5% and 12%, which is slightly conservative but in general agreement with the 95% confidence intervals.¹ The Type II error is less than a fraction of a percent and statistically negligible. The trade-off between Type I and II error can be adjusted in an *ad hoc* manner by changing the critical point in hypothesis test (5.9). Figure 6.4 shows a large gap between damaged and undamaged

¹We expect to see at least 5% error for a properly constructed confidence region/interval.

Table 6.4: Statistical coverage for each damage case expressed as a percentage (the percentage of null hypothesis results). Story levels that experience damage are shown in bold.

Story	\mathcal{D}_0	\mathcal{D}_1	\mathcal{D}_2
1	95.06	94.72	93.94
2	89.93	90.69	90.72
3	87.41	0.00	0.00
4	92.00	92.49	91.73
5	88.57	88.41	87.79
6	89.83	89.85	90.65
7	92.79	92.21	92.38
8	89.16	89.25	0.03
9	92.88	93.76	92.91
10	91.65	91.34	91.36

boxes of identified stiffness, which indicates that it would be possible to create an *ad hoc* hypothesis test with perfect damage detection performance.

6.4 Discussion

The results of this study are interpreted in terms of the estimator performance and its suitability for damage detection. Results will be interpreted with close attention paid to the default case of zero bias and 30 dB SNR.

6.4.1 Estimator Performance

The substructure identification estimator is found to be a generally unbiased and efficient estimator for the story-level stiffness. This conclusion is supported by decreasing error variance and vanishing bias as the measurement noise is decreased. A single exception is the first story which converges to a biased estimate. The author does not have a



Figure 6.4: Box plot of identified stiffness parameter for different damage scenarios. The quantiles shown are 2.5%, 25%, 50%, 75%, and 97.5%.

satisfactory explanation for this behavior but notes that it consistently effects the first story in structures of varying numbers of stories.

The error prediction measure, introduced in Section 4.4, successfully predicts the best and worst performing stories as measured by identification error variance. The prediction measure finds that the lowest performing story should be the eighth story, which indeed has the highest error variance. Likewise, the first floor is predicted to have the highest performance, which identically has the lowest error variance. Heteroskedasticity is encountered in substructure identification because the residual variance is not identically distributed. This behavior is the result of the substructure estimator having higher system response at frequency values near the interstory natural frequency. This behavior can be directly observed in Figure 6.5 for the third story. The observed heteroskedasticity (*i.e.*, non-uniform residual variance by frequency) is expected and does not have a large effect on identification performance. However, it does contribute to error in the computation of confidence intervals. Thus, heteroskedasticity is the leading explanation for the difference in statistical coverage for different confidence intervals. While the presence of heteroskedasticity increases Type I error, it is the author's opinion that the performance of expected Fisher Information confidence interval (computed with $\hat{\mathbf{V}}_{e}$) is sufficient and does not warrant more intrusive measures to remove heteroskedasticity.

Finally, the discussion of damping has been ignored and performance results have focused on stiffness. This choice was deliberate as damping remains difficult to identify in lightly damped structures. This result is encountered in this study and, as such, damping is treated as a nuisance regression parameter that is included to ensure proper stiffness identification. This decision results in no loss of generality because damage detection is wholly determined by stiffness within the scope of this study. Furthermore, by using the identified stiffness parameter in a confidence interval hypothesis test, poor identification of the damping parameter is excluded.



Figure 6.5: Plot showing the mean of H_{EST} and the standard deviation of the residual, as a function of frequency, for various levels of SNR, for the third story.

6.4.2 Damage Detection Regime

Substructure identification is robust to common sources of uncertainty. The first source of uncertainty considered herein was bias error in the previous story stiffness. Substructure identification performed consistently within a small range of bias error. Moreover, error variance does not increase with previous story bias error. This is especially important in the context of damage detection as precision is valued over accuracy. Stated differently, as long as the same baseline is used, low error variance enables damage detection in the presence of initial bias error.

The second source of uncertainty considered herein was measurement noise. This represents a common uncertainty source in SHM applications and substructure identification performed sufficiently within moderate levels of measurement noise (20–30 dB).

Moreover, because substructure identification is an efficient estimator, its error variance can be decreased by using better sensors with higher SNR.

Within the specific context of damage detection, substructure identification satisfactorily detects and localizes damage. This occurs with minimal Type I error and nearly nonexistent Type II error. Furthermore, false positives can be minimized for a specific application by adjusting the critical point of the hypothesis test.

Substructure identification presents itself as an improvement over typical modal methods in the context of damage detection. First, substructure identification decreases model complexity by identifying only a subset of the structure's dynamics. Second, substructure identification is a decentralized algorithm that only requires local measurement of vibration response. This allows for implementation in a wireless network of smart sensors. Third, substructure identification provides inherent damage localization by considering only one area of the structure at a time.

Chapter 7

Controlled Substructure Identification

Substructure identification of a shear building results in varying levels of identification performance for different substructure–structure combinations. Previously, it was demonstrated that the identification of a single story in a uniform shear building can admit different levels of identification performance as measured by identification root mean square error (RMSE) (DeVore and Johnson, 2011; Zhang and Johnson, 2012a). This behavior is predicted by low levels of interstory acceleration response at the story to be identified. This finding motivates research to find ways to temporarily increase the interstory acceleration response during identification to improve identification performance (Zhang and Johnson, 2012b).

This chapter describes the development and simulation of a feedback control regulator that is designed to improve substructure identification performance of a single story. The controller is designed as full state feedback control law using nonlinear optimization. The controller is then combined with an optimal state estimator to form the feedback regulator. The regulator takes measured acceleration signals as inputs and returns a control force as an output. The control force is realized by a control device with static unity



Figure 7.1: Block diagram of system with regulator.

gain. Acceleration time histories are generated and used for substructure identification to determine the identification performance. A simple block diagram of the controlled system is shown in Figure 7.1.

First, this chapter describes several norms useful for multi-objective control design. Second, the uncontrolled system is presented as a linear operator with a state space representation. Third, the control design procedure is described and the performance of several different controllers is compared. Next, an observer is designed for control with various measurement scenarios and the identification performance is described. Finally, the implementation using object oriented programming (OOP) is described.

7.1 Mathematical Preliminaries

This section defines the underlying assumptions and describes several useful mathematical properties for control design. The building is treated as a linear operator, which is consistent with the assumptions of low excitation levels. Multi-objective control design requires several signal and operator norms, which are defined herein using a state space representation.

The input signal **u** is a measurable function from the L_2 Lebsegue space defined on a suitable measure space $(\Omega, \mathfrak{F}, \lambda)$ with the 2-norm $\|\cdot\|_2$ defining the topology. The measure space is taken as a probability space so that **u** is understood to be a vector-valued, zero-mean, Gaussian random process. The L_2 signal norm is given by

$$\|\mathbf{u}\|_{2}^{2} = \int_{\Omega} \mathbf{u}^{\mathsf{H}} \mathbf{u} \, d\lambda = tr\left\{\mathbb{E}\left[\mathbf{u}\mathbf{u}^{\mathsf{H}}\right]\right\} = tr\left\{\mathbf{K}_{\mathbf{u}}\right\}$$

where λ is the probability measure; $(\cdot)^{H}$ implies the complex conjugate transpose; $\mathbb{E}[\cdot]$ is the expectation operator; and $\mathbf{K}_{\mathbf{u}}$ is the covariance matrix of the zero-mean, Gaussian random process \mathbf{u} .

A linear operator representing the building's dynamics can be introduced. The operator *T* maps the input signal to the output signal and is defined as $T : L_2 \mapsto L_2$. Thus, an output signal is defined by $\mathbf{y} = T \mathbf{u}$. Without loss of generality, *T* can be represented as a multiple input, multiple output (MIMO), linear time-invariant (LTI) system with state space representation

$$T := \begin{cases} \dot{\mathbf{x}} = \mathcal{A}\mathbf{x} + \mathcal{B}\mathbf{u} \\ \mathbf{y} = \mathcal{C}\mathbf{x} + \mathcal{D}\mathbf{u} \end{cases}$$

where **u** is the vector-valued input signal, **x** is the state vector, and **y** is the vector-valued output signal. \mathcal{A} is the state transition matrix, \mathcal{B} is the input matrix, \mathcal{C} is the output matrix, and \mathcal{D} is the feedthrough matrix. T also has an intrepretation as a transfer function defined as $T(s) = \mathcal{C}(s\mathbf{I} - \mathcal{A})^{-1}\mathcal{B} + \mathcal{D}$.



Figure 7.2: Block diagram of uncontrolled system.

The collection of linear operators T is a Hilbert space H_2 with an operator norm defined as,

$$||T||_2^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} tr \left\{ T^{\mathsf{H}}(j\omega)T(j\omega) \right\} d\omega$$

where $\left(\cdot\right)^{\mathsf{H}}$ is the complex conjugate transponse of a matrix.

It is not difficult to show that when $\mathcal{D} = 0$, $||T||_2^2 = tr \{\mathcal{C} \mathbf{X}_c \mathcal{C}^T\}$ where \mathbf{X}_c is the controllability grammian and is the postitive definite solution to the Lyapunov equation

$$\mathcal{A}X_{c} + X_{c}\mathcal{A}^{\mathsf{T}} + \mathcal{B}\mathcal{B}^{\mathsf{T}} = 0$$
(7.1)

Equivalently, the operator norm can computed in terms of the observability grammian. The interested reader is referred to Burl (1999).

The H_{∞} norm of the linear operator can be defined as,

$$||T||_{\infty} = \max_{\omega} \sigma_{\max} [T(j\omega)]$$

where $\sigma_{\max}(\cdot)$ is the maximum singular value of the matrix. This norm can provide a bound on the system gain which is useful for describing disturbance rejection.

7.2 Uncontrolled System

The uncontrolled structure is a 10 story, uniform shear building with properties given in Chapter 3. The input signal is band limited, white noise excitation applied to the structure as ground acceleration \ddot{u}_g . The output signals are the acceleration at each floor including the ground excitation such that $\mathbf{y} = [\ddot{u}_g, \ddot{x}_1, \dots, \ddot{x}_{N_{\text{DOF}}}]^{\mathsf{T}}$. The measured output signals are polluted with measurement noise vector \mathbf{v} .

A linear operator, T^u, corresponding to the uncontrolled structure is defined as,

$$T^{u} := \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{G}\ddot{u}_{g} \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{H}\ddot{u}_{g} + \mathbf{v} \end{cases}$$
(7.2)

with the state space matrices defined as,

$$\begin{split} \mathbf{A} &:= \begin{bmatrix} \mathbf{0}_{N_{\text{DOF}} \times N_{\text{DOF}}} & \mathbf{I}_{N_{\text{DOF}} \times N_{\text{DOF}}} \\ -\breve{\mathbf{M}}^{-1}\breve{\mathbf{K}} & -\breve{\mathbf{M}}^{-1}\breve{\mathbf{C}} \end{bmatrix}_{2N_{\text{DOF}} \times 2N_{\text{DOF}}} & \mathbf{G} := \begin{bmatrix} \mathbf{0}_{N_{\text{DOF}} \times 1} \\ -\mathbf{1}_{N_{\text{DOF}} \times 1} \end{bmatrix}_{2N_{\text{DOF}} \times 1} \\ \mathbf{C} &:= \begin{bmatrix} \mathbf{0}_{1 \times N_{\text{DOF}}} & \mathbf{0}_{1 \times N_{\text{DOF}}} \\ -\breve{\mathbf{M}}^{-1}\breve{\mathbf{K}} & -\breve{\mathbf{M}}^{-1}\breve{\mathbf{C}} \end{bmatrix}_{(N_{\text{DOF}} + 1) \times 2N_{\text{DOF}}} & \mathbf{H} := \begin{bmatrix} 1 \\ \mathbf{0}_{N_{\text{DOF}} \times 1} \end{bmatrix}_{(N_{\text{DOF}} + 1) \times 1} \end{split}$$

where $\mathbf{\check{M}}$ is the diagonal mass matrix; $\mathbf{\check{K}}$ and $\mathbf{\check{C}}$ are the positive definite stiffness and damping matrices; and N_{DOF} is the number of stories. A block diagram of the system is shown in Figure 7.2.



Figure 7.3: Block diagram of controlled system.

Measurement noise is additive Gaussian white noise provided by \mathbf{v} . The noise is assumed to be zero-mean, statistically independent, and applied to each output channel. The noise covariance matrix is

$$\mathbf{K}_{\mathbf{v}} = \mathbb{E}\left[\mathbf{v}\mathbf{v}^{\mathsf{T}}\right] = \frac{\|\ddot{u}_{g}\|_{2}^{2}}{(\mathrm{SNR})^{2}} \mathbf{I}_{(N_{\mathrm{DOF}}+1)\times(N_{\mathrm{DOF}}+1)}$$

where $\|\ddot{u}_g\|_2$ is the standard deviation of the input ground acceleration, taken in this study to be 0.1 m/s², and SNR is the signal to noise ratio, taken in this study to be 30 dB.

7.3 Controlled System

A full state feedback controller is designed for the testbed and its performance is discussed within this section. First, the operator is defined for a full state controller allowing for a consistent description. Next, the control design problem is defined in terms of the poles of the closed loop system. Following, linear constraints on the search space are used to meet stability and damping targets while nonlinear constraints are used to ensure realistic performance. Finally, the performance of several different feedback control laws are compared using Monte Carlo simulation to determine the substructure identification performance. Furthermore, the damage detection performance is demonstrated using a direct comparison between undamaged and damaged scenarios.

7.3.1 Full State Feedback Controller

A full state feedback controller is defined by multiplying a feedback gain matrix **K** by the state vector and applying the resulting signal as a control force $\mathbf{u}_c = -\mathbf{K}\mathbf{x}$. Through convention, feedback control is applied as negative feedback and that convention is respected within this work. The block diagram of the controlled system is shown in Figure 7.3.

Using the feedback gain matrix, the controlled system can be defined as a linear operator

$$T^{c} := \begin{cases} \dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{G}\ddot{u}_{g} \\ \mathbf{y} = (\mathbf{C} - \mathbf{D}\mathbf{K})\mathbf{x} + \mathbf{H}\ddot{u}_{g} + \mathbf{v} \end{cases}$$
(7.3)

Without loss of generality, the control force state space matrices can be defined for a shear building with an ideal AMD installed on the roof. Thus, the control input matrix **B** and control feedthrough matrix **D** are defined,

$$\mathbf{B} := \begin{bmatrix} \mathbf{0}_{(2N_{\text{DOF}}-1)\times 1} \\ -1/m_{N_{\text{DOF}}} \end{bmatrix}_{2N_{\text{DOF}}\times 1} \qquad \mathbf{D} := \begin{bmatrix} \mathbf{0}_{N_{\text{DOF}}\times 1} \\ -1/m_{N_{\text{DOF}}} \end{bmatrix}_{(N_{\text{DOF}}+1)\times 1}$$

where $m_{N_{\text{DOF}}}$ is the mass of the top floor of the building and N_{DOF} is the number of stories. The other state space matrices are the same as defined in the previous section.

7.3.2 Control Design

The control gain matrix is designed using nonlinear optimization subject to linear and nonlinear constraints. A nonlinear objective functional is calculated for each sample point and the optimization algorithm continues until a local minimum is encountered. This section describes the objective functional and search space while the succeeding sections describe the constraints.

The objective functional is selected to increase the interstory acceleration response of the closed loop system within the identification bandwidth. Therefore, the functional *J* takes the form,

$$J := \int_{\omega_{1}}^{\omega_{u}} \left| W(j\omega) \left[T_{i}^{c}(j\omega) - T_{i-1}^{c}(j\omega) \right] \right| d\omega$$
(7.4)

where $W(j\omega)$ is the weighting function taken to be the model function H_{MOD} ; ω_{l} and ω_{u} are the lower and upper bounds of the identification frequency bandwidth, respectively; and $T_{i}^{c}(j\omega)$ is the transfer function of the closed loop operator for a specific control gain evaluated at the frequency ω for the *i*th floor acceleration output. *J* is understood to be the weighted integral of the interstory acceleration response, the same as provided in Section 4.4.

With the state space matrices known, *J* can vary with different choices of **K**. Previous studies (Zhang and Johnson, 2012b) used the various entries of the control gain matrix as the search space. In this study, the closed loop poles are used as the search space so that *J* is understood to be a functional mapping of the search space to a positive real value $J : \mathbb{R}^{2N_{\text{DOF}}} \mapsto \mathbb{R}^+$. The search space has a one-to-one correspondence with the various

control gain entries while providing unique computational benefits by specifying some constraints as linear constraints. This will be discussed further in the next section.

The uncontrolled system is stable and lightly damped implying that each pole has a conjugate pair. Using the closed loop poles, the parameter space can be represented as the real component and the absolute value of the imaginary component of the pole. Therefore, for each pole and its conjugate, $(p_i, p_i^*) \rightarrow (a_i, b_i)$ where $a_i = \Re \{p_i\}$ and $b_i = |\Im \{p_i\}|$. Thus, a N_{DOF} building will have a $2N_{\text{DOF}}$ dimensional vector as the design space, $\mathbf{q} \in \mathbb{R}^{2N_{\text{DOF}}}$ such that $\mathbf{q} = [a_1, \ldots, a_{N_{\text{DOF}}}, b_1, \ldots, b_{N_{\text{DOF}}}]^{\mathsf{T}}$.

For each evaluation of the objective function, \mathbf{q} is used to compute a full-state feedback controller \mathbf{K} with the specified closed loop poles. Then, the closed loop operator is found and used to compute the weighted integral of the interstory acceleration response at a given floor. This value is used by the optimization algorithm to maximize response. If \mathbf{q} is from an infeasible set, \mathbf{K} will be returned as a matrix of zeros.

The initial condition for optimization, \mathbf{q}_0 , is selected to be the uncontrolled poles of the structure. This is found to have a stabilizing effect on control design and yields a consistent controller. Previously, a randomized set of poles grouped around the interstory natural frequency was used. However, the randomized poles would often be from an infeasible set which prevents the optimization algorithm from converging.

The final solution \mathbf{q}_f is found by,

$$\mathbf{q}_{f} = \arg \min_{\mathbf{q}} \qquad -J(\mathbf{q})$$

subject to: $\mathbf{M}\mathbf{q} + \mathbf{m} \le 0$ (7.5)
 $c_{NL}(\mathbf{q}) \le 0$

110



Figure 7.4: Feasibility region for closed-loop poles.

where **M** and **m** define the linear constraints and $c_{NL}(\cdot)$ is a function that defines the nonlinear constraints. By minimizing the negative of the objective functional, the interstory acceleration response is increased in the frequency band where $W(j\omega)$ is large and the identification performance should be improved.

7.3.3 Linear Constraints

Two linear constraints are included in the control design. For each closed loop pole of the system p_i specified by (a_i, b_i) , the following linear constraints are applied.

$$a_i \le \sigma^{\mathsf{c}}$$
 (7.6a)

$$a_i + \zeta^{\mathsf{c}} b_i \le 0 \tag{7.6b}$$

where σ^{c} is the pole constraint and ζ^{c} is the minimum damping ratio for each pole. The first constraint ensures that the closed loop system is stable and achieves a minimum level of settling time for each pole. The second constraint ensures that the damping of each mode is greater than ζ^{c} . These constraints on pole location are shown in Figure 7.4.

The linear constraint inequalities expressed in (7.6) can be constructed in a form suitable for the optimization problem defined in (7.5). For a given realization of **q**, the linear constraints are satisfied if $\mathbf{Mq} + \mathbf{m} \leq 0$ with **M** and **m** defined as

$$\mathbf{M} := \begin{bmatrix} \mathbf{I}_{N_{\text{DOF}} \times N_{\text{DOF}}} & \mathbf{0}_{N_{\text{DOF}} \times N_{\text{DOF}}} \\ \mathbf{I}_{N_{\text{DOF}} \times N_{\text{DOF}}} & \zeta^{c} \mathbf{I}_{N_{\text{DOF}} \times N_{\text{DOF}}} \end{bmatrix} \qquad \mathbf{m} := \begin{bmatrix} -\sigma^{c} \mathbf{1}_{N_{\text{DOF}} \times 1} \\ \mathbf{0}_{N_{\text{DOF}} \times 1} \end{bmatrix}$$

For each closed loop pole, $(a_i, b_i) = (-\zeta_i \omega_i, \omega_i \sqrt{1 - \zeta_i^2})$, the inequality in (7.6b) implies that

$$-\zeta_{i}\omega_{i} + \zeta^{c}\omega_{i}\sqrt{1-\zeta_{i}^{2}} \leq 0$$
$$\zeta^{c}\omega_{i}\sqrt{1-\zeta_{i}^{2}} \leq \zeta_{i}\omega_{i}$$
$$\zeta^{c} \leq \frac{\zeta_{i}}{\sqrt{1-\zeta_{i}^{2}}}$$

where the natural frequency and damping ratio of each pole is ω_i and ζ_i , respectively. Thus, ζ_i is greater than ζ^c but is an unconservative bound for an underdamped system because $\sqrt{1-\zeta_i^2} < 1$. For damping constraint values used herein, the difference is approximately 2% of the constraint value. Therefore, a damping constraint safety factor of 1.05 would be sufficient to ensure minimum damping levels, though a safety factor is not used in this study.

7.3.4 Nonlinear Constraints

To further constrain the control design, nonlinear constraints are used. Three different nonlinear constraints are considered: control force RMS, H_2 constraint, and H_{∞} constraint. The first two constraints can be broadly understood as energy constraints and the last constraint is a disturbance rejection constraint. The three constraints are described herein.

7.3.4.1 Control Force Constraint

The control force constraint ensures that, given an input ground acceleration signal of certain energy, the control force energy will not exceed a certain level. This constraint does not provide an upper bound to the control force commanded, which can result in a saturated control force signal in practice.

The energy of the control force is determined by finding a solution to the L_2 norm of the control force signal $||u_c||_2$. This is given by multiplying the control force gain matrix by the controllability grammian. Thus,

$$\|u_{\mathbf{c}}\|_2^2 = \mathbf{K} \mathbf{X}_{\mathbf{c}} \mathbf{K}^{\mathsf{T}} \le f^{\mathsf{c}}$$

where the controllability grammian \mathbf{X}_c is found by solving the Lyapunov equation given in (7.1) using the state space matrices of T^c and f^c is the force constraint.

In this study, $||u_c||_2$ is computed when the building is subjected to a white noise ground acceleration signal RMS of 0.1 m/sec² (*i.e.*, $||\ddot{u}_g||_2 = 0.1 \text{ m/s}^2$). The control force norm is expressed as a percentage of the total weight of the building; the default constraint is 2%.

7.3.4.2 System Energy Constraint

The system energy constraint ensures that the controller does not unduly amplify the output energy. This constraint is understood as a H_2 constraint and requires that the 2-norm of the operator be below a certain threshold. The constraint is given by,

$$\|T^{\mathsf{c}}\|_{2} = \sqrt{tr\left\{\mathbf{C}\mathbf{X}_{\mathsf{c}}\mathbf{C}^{\mathsf{T}}\right\}} \le h_{2}^{\mathsf{c}}$$

where h_2^c is the system energy constraint.

This constraint is applied to the output acceleration signals of the building only and omits the ground acceleration (y_i for $i = 2, ..., N_{DOF} + 1$). This is purposeful because the H_2 norm of an operator is infinite for a linear operator with non-zero feedthrough (*i.e.*, $\mathcal{D} \neq 0$). By removing the ground acceleration from the output vector, the operator 2-norm can be found. Moreover, this better reflects the physical nature of the system.

7.3.4.3 Disturbance Rejection Constraint

The disturbance rejection constraint ensures that a bounded input will provide a bounded output below a certain threshold. This constraint is understood as a H_{∞} constraint applied such that $||T^{c}||_{\infty} \leq h_{\infty}^{c}$. Here, h_{∞}^{c} is the constraint, which is implemented by the MATLAB command norm for the H_{∞} case.

7.3.5 Comparison

Different controllers can be developed by specifying different constraints for the linear and nonlinear constraints developed previously. In this study, several controllers are

B]
0
0
0
0
0
-
0
0

Table 7.1: Design specifications of various controllers as specified by various constraints. Note: the force constraint is specified as a percentage of the total weight of the structure.

Table 7.2: Design results of various controllers compared to the uncontrolled case. Note: the force constraint is specified as a percentage of the total weight of the structure.

	J	$\max_i \sigma_i$	$\min_i \zeta_i$	$\ u_{c}\ _{2}$	$ T^{c} _{2}$	$ T^{c} _{\infty}$	$\max_i \ \delta_i\ _2$	$\max_i \ \ddot{x}_i\ _2$
Name	[·]	[rad/s]	[%]	[%]	[dB]	[dB]	[mm]	[m/sec ²]
UNC	0.30	-0.10	1.49	0.00	30.31	39.78	4.82	1.40
CT00	1.82	-5.10	11.63	1.91	27.26	18.01	1.02	1.31
CT01	13.40	-5.00	10.17	5.00	39.66	33.79	2.26	4.47
CT02	0.41	-4.71	14.23	1.89	21.22	12.12	1.56	0.46
CT03	3.29	-5.40	10.43	1.93	28.67	19.88	1.45	1.27
CT04	4.57	-5.00	10.19	2.00	30.53	24.10	1.25	1.49
CT05	_	-5.00	9.95	_	101.11	98.25	_	_
CT06	4.26	-5.04	10.34	1.98	30.09	22.73	1.49	1.46
CT07	4.93	-2.00	9.97	2.00	31.38	25.81	1.59	1.73

compared; their constraint specifications are provided in Table 7.1. The default controller CT00 represents a reasonable selection of constraints likely encountered in practice. The other controllers are deviations from this default scenario to elucidate the effects of the constraints on control design.

Using the constraint specifications provided in Table 7.1, each controller is designed using the procedure described in Section 7.3.2. Then, the control design results are tablulated in Table 7.2. This table includes the various constraint norms and also shows the maximum interstory deflection δ_i and maximum story acceleration \ddot{x}_i norms. The results of the control design can be visualized in Figures 7.5 and 7.6. These figures show the interstory acceleration response of the eighth story, which is the story response that is to be increased.

The control design provides several interesting results. First, CT02 is infeasible because $||T^c||_2$ violates the constraint of 20 dB. Second, the damping constraint provides an unconservative bound that is reflected in some of the controllers reporting poles with the damping constraint slightly violated. Third, CT00 and CT06 satisfy each other's design constraints but admit different control gain matrices. This means that the optimization proceedure is non-convex and will admit local minima.

An exception to the design of the controllers is CT05, which is a controller designed with linear constraints only. This design is unlikely to be implemented due to its extremely large system energy amplification and control force requirements. It should only be regarded as an interesting academic exercise of control design with linear constraints and not as a feasible controller.

The controllers are further evaluated by determining their substructure identification performance. MCS is used with 10,000 samples and the eighth story stiffness is identified using time histories generated using the various controllers. A damage scenario is provided that evaluates the damage detection performance and demonstrates the controller robustness. The results are summarized and compared against the uncontrolled system in Table 7.3.



Figure 7.5: Eighth story interstory acceleration response for various controllers defined in Table 7.1. The center of the identification bandwidth is shown as a dashed line.



Figure 7.6: Eighth story interstory acceleration response for various controllers defined in Table 7.1. The center of the identification bandwidth is shown as a dashed line.

The results of substructure identification show that the controllers generally have significantly better identification performance as indicated by decreased bias error and decreased variance. The results match the performance prediction provided by the weighted integral *J* of interstory acceleration response.

Four of the controllers do not have values for the identification statistics for the damage scenario because the system was unstable. This behavior was caused because the controlled system was not robust to uncertainties in the stiffness parameter. The most aggressive controllers (larger control gain) generally were the least robust controllers to stiffness changes. Thus, there appears to be a (not unexpected) trade-off between control authority and system robustness.

CT00, CT03, and CT07 have comparable levels of identification performance and are a significant improvement over the uncontrolled system. CT00 is selected as the default because it has a smaller control gain than CT03 and has its maximum pole further in the left half plane than CT07. These two properties indicate that CT00 will be more stable under varying damage scenarios and will admit better observer performance.

Table 7.3: Statistics of substructure identification of the eighth story stiffness presented as percentage of the nominal stiffness value. The damaged case corresponds to a decrease in stiffness of 5%.

	Undamaged		Dama	nged
	Mean	STD	Mean	STD
UNC	-0.29	0.61	-5.33	0.55
CT00	-0.01	0.07	-5.01	0.05
CT01	-0.00	0.01	_	_
CT02	-0.41	0.32	-5.35	0.31
CT03	0.01	0.04	-5.00	0.03
CT04	0.00	0.03	_	_
CT05	-0.00	0.00	_	_
CT06	0.00	0.03	_	_
CT07	0.00	0.03	-5.00	0.03



Figure 7.7: Block diagram of observed system.

7.4 Observed System

The control law designed in the previous section requires full state measurement, which is unlikely encountered in practice. Therefore, an observer is used to estimate the states of the system using measured responses. This section will describe the design and performance of the observed system for various observers. Different observers are created by changing which sensor channels are measured. First, an observer gain matrix is designed using the Kalman filter. Second, various observers are used in conjuction with CT00 to determine the substructure identification performance.

7.4.1 Observer Design

An observer is designed to estimate the states of the system for feedback control from a measured subset $\tilde{\mathbf{y}} = [y_{i_1}, y_{i_2}, ...]^T$ of the acceleration responses. The observer is characterized by an observer gain matrix **L**. Taken together, the observer and control gain matrices form the feedback control regulator that is shown in Figure 7.1. The resulting system operator is defined as,

$$T^{o} := \begin{cases} \begin{pmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{x}} \end{pmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}\mathbf{K} \\ \mathbf{L}\tilde{\mathbf{I}}\mathbf{C} & \mathbf{A} - \mathbf{B}\mathbf{K} - \mathbf{L}\tilde{\mathbf{I}}\mathbf{C} \end{bmatrix} \begin{pmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{pmatrix} + \begin{bmatrix} \mathbf{G} \\ \mathbf{L}\tilde{\mathbf{I}}\mathbf{H} \end{bmatrix} \ddot{u}_{g} + \begin{bmatrix} \mathbf{0} \\ \mathbf{L}\tilde{\mathbf{I}} \end{bmatrix} \mathbf{v} \\ \mathbf{y} = \begin{bmatrix} \mathbf{C} & -\mathbf{D}\mathbf{K} \end{bmatrix} \begin{pmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{pmatrix} + \mathbf{H}\ddot{u}_{g} + \mathbf{v} \end{cases}$$
(7.7)

where $(\mathbf{x}^{\mathsf{T}}, \mathbf{\hat{x}}^{\mathsf{T}})^{\mathsf{T}}$ is the augmented state vector including the estimated state vector $\mathbf{\hat{x}}$ and $\mathbf{\tilde{I}}$ contains the rows of an identity matrix corresponding to the measured accelerations defined by $[\mathbf{\tilde{I}}]_{k,l} = \delta_{i_k,l}$ using the Kronecker delta. The block diagram is shown in Figure 7.7.

The observer is designed using the Kalman filter which provides the optimal estimate of the state variable (Mendel, 1995). The design is implemented using the MATLAB command kalman. This command takes as input, the uncontrolled system expressed as a state space model, the input ground acceleration RMS ($||\ddot{u}_g||_2 = 0.1 \text{ m/sec}^2$), and the sensor noise covariance matrix $\mathbf{K}_{\mathbf{v}}$. The output gives the observer gain matrix \mathbf{L} . Once the observer gain matrix is designed, the observed system operator T^{o} is constructed as defined in (7.7).

Often, it is not feasible that each sensor is available for state estimation, so this study will design observers with incomplete sensor measurement. This is accomplished by restricting the measured sensors such that the rows of **C**, **D**, and **H** are selected corresponding to the measured sensors using **I**. Then the observer is designed as described.

7.4.2 Comparison

In this study, several observers are compared using different sets of sensors. The first scenario OB00 is of full acceleration measurement, which serves as the default case. The subsequent scenarios use different sets of sensors measured. The various performance norms are shown in Table 7.4. Likewise, the eighth story interstory acceleration response is shown in Figure 7.8 for the various observers. Substructure identification performance is demonstrated for each observer in Table 7.5.

The results show that substructure identification is successful under each of the observers. The first three observers exhibit performance that is nearly identical to the full state feedback controller. OB03 and OB04 do not measure the input ground motion and subsequently introduce additional dynamics into the system. OB03 has slightly worse identification performance and OB04 has slightly better identification performance compared to CT00. This behavior is predicted by the weighted integral *J* of interstory acceleration response.

	Obs. Vec.	J	$\max_i \sigma_i$	$\min_i \zeta_i$	$ u_{c} _{2}$	$ H _{2}$	$\ H\ _{\infty}$	
Name		[·]	[rad/s]	[%]	[%]	[dB]	[dB]	
UNC	-	0.30	-0.10	1.49	0.00	30.31	39.78	
CT00	full state	1.82	-5.10	11.63	1.91	27.26	18.01	
OB00	full accel.	1.82	-4.36	11.63	1.91	27.26	18.06	
OB01	[0,7,8,9,10]	1.82	-4.26	11.63	1.91	27.26	18.06	
OB02	[0,10]	1.82	-3.52	11.00	1.91	27.26	18.05	
OB03	[7,8,9,10]	1.54	-5.10	11.27	1.56	26.28	18.29	
OB04	[10]	2.27	-5.10	11.63	1.63	26.62	17.57	

Table 7.4: Design results of various observers. The first columns refer to which story-level acceleration sensors are used with zero corresponding to the ground acceleration. $\| Obs. Vec. \| I \| max_i \sigma_i \| min_i \zeta_i \| \| u_0 \|_2 \| \| H \|_2$

Table 7.5: Statistics of controlled identification with observers of the eighth story parameters presented as percentages.

	Undan	naged	Dama	nged
	Mean	STD	Mean	STD
UNC	-0.29	0.61	-5.33	0.55
CT00	-0.01	0.07	-5.01	0.05
OB00	-0.01	0.07	-5.01	0.06
OB01	-0.00	0.07	-5.00	0.05
OB02	-0.01	0.07	-5.01	0.06
OB03	0.23	0.09	-4.83	0.07
OB04	0.00	0.05	-5.00	0.04



Figure 7.8: Eighth Story Interstory Acceleration Response for various observers.

7.5 Implementation

The simulations are implemented using object oriented programming (OOP) within MATLAB. This implementation abstracts the various operators into different classes which provides several benefits: modularity, inheritance, and security. To understand this study's implementation, a brief description of OOP is provided along with the unique benefits to this study. Finally, a description of the workflow for the simulation is provided. The class diagram describing the particular implementation is given in Figure 7.9.

OOP is a paradigm that seeks to abstract data types into objects with predefined properties and methods. The form of the data stored within an object is specified in a class file. Also included within the class file is a series of methods that can operate on the data stored within an object. Methods can also incorporate outside data input while providing output. Class inheritance can be provided where a subclass inherits the properties and methods of a superclass.

Within this study each different operator (T^u , T^c , and T^o) is implemented as its own class. Starting with the uncontrolled operator, an object is defined and then used for the input for the next operator. Each subsequent operator adds another layer of properties and methods. This abstraction improves modularity within the code by separating methods and properties that are entirely contained within a certain object. As such, modularity improves code maintanence.

The second unique benefit to OOP, within this study, is that inheritance promotes code reuse. The substructure identification and time history simulation code is contained within the FRFSubID class. This means that the properties and methods are not duplicated in different simulations which provides robust code. Furthermore, the subclasses that inherit FRFSubID implement an interface defined by LinearSystem that allows proper behavior.

Another unique benefit of OOP is code security. Different properties can be marked as private or protected, which means that their value cannot be changed by the user or even by another method (if properly specified). This helps protect against bugs in projects with a large codebase. In this study, various properties are marked as protected or private for this reason (refer to Figure 7.9 for details).

A unique benefit to MATLAB's implementation of OOP is that properties can be defined as dependent. A dependent property is one whose value is calculated each time that it is called. This behavior is exploited for properties that are dependent on the stiffness of the building (**A** and **C**). The stiffness can be changed at will by modifying a public property kdamage. This allows for simulation of damage after the controller and/or observer was designed for the undamaged case.

The basic workflow of the simulation is to construct a ShearBuilding object. Then, a ControlledShearBuilding object is constructed using the previous object as input along with certain control specifications. Finally, an ObservedControlledShearBuilding object is constructed using the previous object as input along with specifications of which channels are measured. After each object is constructed, substructure identification is simulated by calling the identify_story method, which simulates a time history (create_time_history), computes the reduced order model (compute_ROM_FRF), and then, finally identifies the stiffness (identify_ROM).

In this implementation, great care was taken to improve the efficiency of the code. This enabled the author to run more Monte Carlo simulations and improve control design.

127

Code efficiency was improved by loading data into memory and using anonymous functions. This was achieved through various methods including identify_story and control_design. A noticable improvement in execution and optimization time was observed from previous work.



Figure 7.9: Class diagram of implemented methods.

Part III

Experimental Results

Chapter 8

Bench Scale Structure

Experimental testing of substructure identification is performed on a bench scale, two story, flexible structure at the University of Southern California. This experiment is designed to validate controlled substructure identification using a passive control scenario. This chapter is organized to first describe the experimental apparatus, including the structure, shake table, and sensors, used in the experiment. Next, the experimental procedure is developed where different structural configurations are described. Finally, results are presented that find that substructure identification precision can be increased using structural control.

8.1 Experimental Apparatus

The proposed experiment is implemented at bench scale with two basic components: a Quanser shaking table and a two story flexible structure. The shake table is capable of producing a maximum acceleration of 2.5 g with a 25 lb payload. The table is commanded by a control computer that performs both feedback control and data acquisition at 1000 Hz. Using the control computer, an arbitrary table acceleration time history can be achieved,
in which constant acceleration response of the table is observed within the bandwidth of the structure (2–8 Hz). The shake table with the structure installed is shown in Figure 8.1.



Figure 8.1: Bench scale structure and shake table.

The two story structure is specifically designed to deform in a manner consistent with a 2DOF shear building. Seismic mass is provided by Plexiglas plates installed at each story level. In addition, these plates provide a rigid floor connection that ensures that the structure behaves as a shear building. Lateral stiffness is provided by thin sheet metal plates and linear spring braces. The sheet metal provides exceptional out of plane

k_i	572 N/m
Ci	0.29 N/(m/s)
m_i	0.934 kg
$\omega_{0,i}$	3.94 Hz

Table 8.1: Nominal story parameters for bench scale structure

stiffness while giving very little in plane stiffness. Linear springs are installed in a \times -brace configuration for each story. These springs are configurable and can be changed to alter the dynamics of the structure.

Before conducting substructure identification, the equivalent mass, damping, and stiffness parameters for a characteristic story are identified. First, the structure is disassembled and the components are weighed to determine their mass. The mass matrix is assembled using lumped mass consistent with a 2-DOF ROM. Then, the stiffness is identified by assembling the components into a single story structure and subjecting the resulting structure to swept sine excitation. The frequency of maximum response is extracted and used to compute the stiffness parameter using the measured mass. Finally, the damping parameter is found by displacing the one story structure and measuring the free vibration response. The log decrement method is used to compute the damping parameter using the average of five periods. The identified SDOF values, shown in Table 8.1, are assumed to be the same as the 2-DOF parameters.

It is noted that the structure has very low levels of damping, with equivalent modal damping less than 1%. This implies that it will be difficult to accurately identify the damping parameter. Therefore, only the stiffness parameter will be estimated in the subsequent substructure identification tests. During these tests, the damping parameter is



Figure 8.2: Bench scale performance index for various control strategies

treated as a nuisance regression term and allowed to vary between 0.5% and 1.5%. In most identification runs, the damping parameter converged to either the lower or upper bound.

While these values present a starting point for control design, they should not be accepted as the true values. It is noted that when the structure was reassembled to the two story configuration, the first story appeared to be softer, with apparent stiffness lower than identified and given in Table 8.1. This is attributed to the second story mass adding a compressive load to the first floor side walls, creating P- δ effects that reduce the first story stiffness.

8.2 Control Design

The error analysis results of Section 5.4 motivates controlled substructure identification where the identification accuracy is improved via structural control. Zhang and Johnson (2012b) develop an error analysis, discussed in Section 5.4.2. Here, the error analysis from Zhang and Johnson (2012b) is used to improve the identification accuracy for the bench scale structure.

In this study, simply achievable passive control strategies are used, which limits possible control strategies. This selection was made to reduce the complexity of the experiment and because other control strategies were unavailable. Moreover, because it is beneficial to not directly apply control forces to the substructure being identified, the possible control strategies are further restricted to changing the mass of the second story to improve first story identification and changing the mass or stiffness of the first story to improve second story identification.

To test the relative improvement that the control strategies give, a performance function is created. This function seeks to minimize the identification error as given by Zhang and Johnson (2012b). Moreover, the identification error does not have a meaning in an absolute sense so it is necessary to consider the relative improvement to some baseline structure. The selected baseline is the structure identified in the previous section.

The governing performance function is

$$E_{i}^{Z}(\mathbf{L}) = \alpha \int_{\omega_{1}}^{\omega_{N}} \left| W(j\omega) \frac{1}{H_{(\ddot{x}_{i}-\ddot{x}_{i-1}),\ddot{u}_{g}}(j\omega)} \right|^{2} d\omega + (1-\alpha) \int_{\omega_{1}}^{\omega_{N}} \left| W(j\omega) \frac{H_{(\ddot{x}_{i+1}-\ddot{x}_{i}),\ddot{u}_{g}}(j\omega)}{H_{(\ddot{x}_{i}-\ddot{x}_{i-1}),\ddot{u}_{g}}(j\omega)} \right|^{2} d\omega \quad (8.1)$$

135

where *i* is the story-level being identified, **L** is a matrix representing the control strategy applied, $W(j\omega) = \omega_{0,i}^2 \omega^2 / [\omega^2 - 2 * \zeta_i \omega_{0,i} j\omega - \omega_{0,i}^2 \omega^2]^2$ is a weighting function that is sharply peaked at the interstory natural frequency $\omega_{0,i} = \sqrt{k_i / m_i}$, $H_{\ddot{x}_i - \ddot{x}_{i-1}, \ddot{u}_g}(j\omega_n)$ is the FRF from the base motion to the lower interstory acceleration at *i* and $H_{\ddot{x}_{i+1} - \ddot{x}_i, \ddot{u}_g}(j\omega_n)$ is the FRF from the base motion to the upper interstory acceleration at *i*. Note that smaller values of $E_i^Z(\mathbf{L})$ predict less identification error.

The performance index is computed for various structural configurations and then compared to the baseline structure. The value that minimizes $E_i^Z(\mathbf{L})$ for a particular \mathbf{L} will provide the best performance. The performance for various structural configurations is shown in Figure 8.2. From this figure, it is clear that increasing first story stiffness will improve second story identification and decreasing the second story mass will improve first story identification. Therefore, these two control strategies will be implemented.

8.3 Experimental Procedure

Using the results of the previous section, an experimental procedure is designed to test the hypothesis that identification accuracy can be improved by a specifically designed control system. Physically, this is implemented by designing an uncontrolled structure that allows for removal of second story mass and addition of first story stiffness. Thus, the three structural configurations become:

1. Uncontrolled Structure:

The nominal structure is used with an additional mass installed on the roof, which results in the second story mass being 65% larger than that of the nominal structure.

2. Control Structure I:

The additional mass on the second floor is removed resulting in the nominal structure, which should improve first story identification accuracy.

3. Control Structure II

The additional second floor mass is in the structure like the "uncontrolled" structure but additional braces are installed in the first story resulting in the first story stiffness being 180% larger than nominal.

The particular values were selected based on available materials and do not represent the optimal selection. The effects of the two control strategies are shown in Figures 8.3 and 8.4. These figures show that the low interstory acceleration response is increased in both control cases, which will decrease E_i^Z as the interstory acceleration is in the denominator of (8.1).

Each of the structural configurations are tested with ten independent trials. Each of the stiffness and damping parameters are estimated for the uncontrolled configuration but only the first story parameters are estimated for the first control structure and only the second story parameters for the second control structure. This is because the parameters of the other floor changed as a result of the control strategy. This is an important point because the identification of the entire structure would require multiple tests with different configurations. This would be difficult to achieve with purely passive methods; rather, active or semi-active methods would better suited.

For each trial, the structure is subjected to band limited white noise base excitation. The cutoff frequencies are selected to include the bandwidth of the structure (2–8 Hz).



Figure 8.3: Bench scale structure first inter-story acceleration FRF for first control case.

The acceleration responses are recorded for thirty minutes at 1000 Hz and are later downsampled to 100 Hz for the analysis.

Substructure identification is performed by batch processing the data. First, the time histories are arranged in an ensemble with the use of a half-range sine window function and 67% overlap. Then, the cross-power spectral density functions (CPSDs) are estimated using Welch's method with a 2¹² point FFT, selecting the shake table command signal as the reference signal. The command signal is used because it has minimal noise and is highly correlated with the response signals. Finally, the estimated CPSDs are combined and non-linear regression is performed using a LS estimator, consistent with Zhang and Johnson (2012c). The optimal estimate is found with the *Coliny Adaptive Pattern Search*



Figure 8.4: Bench scale structure second inter-story acceleration FRF for second control case.

method using DAKOTA, which is a coordinate-based gradient-free optimization method. This method is found to outperform gradient-based optimization methods due to its robustness against local minima that are more prevalent in physical systems such as this.

8.4 Results

The identified stiffness and damping values are shown visually in Figure 8.5 and statistics are recorded in Table 8.2. By inspecting Figure 8.5 and looking at the inter-quartile range in Table 8.2, the two control structures exhibit tighter grouping in the estimated parameters which indicates improved identification precision. There is an observed



Figure 8.5: Identified stiffness parameters for bench scale experiment. The width of the box shows the IQR and the middle line shows the median.

bias from the nominal values (Table 8.1) which is not troublesome because the overall bias is low and there is no *prima facie* reason to believe the swept-sine estimates are more accurate than the substructure estimates. Further, from an information theoretic perspective, the substructure estimates should be more accurate because they utilize more data (information).

First Story Second Story Median IQR Median IQR [N/m][N/m][N/m][N/m]Uncontrolled 528.04 19.40 545.66 11.23 Control 01 537.45 4.48 n/a n/a Control 02 559.58 5.22 n/a n/a

Table 8.2: Identified stiffness statistics for bench scale experiment

Regardless of the level of bias error, the most important statistic in evaluating a damage detection method is the error variance. These results clearly indicate that the variance is decreased when a control strategy is applied. On the strength of this observation, controlled substructure identification is confirmed effective for this structure.

While passive control methods are simple to implement, they do not represent a feasible full scale solution. Further research is needed to consider active and semi-active control devices, which are more reasonable implementations for controlled substructure identification. However, this particular structure exhibits undesirable dynamic properties due to its extreme flexibility. Moreover, the associated active control device, a second floor mounted AMD, does not have a controlled response with sufficient authority and bandwidth to implement a secondary control law designed to improve identification accuracy.

In conclusion, controlled identification was found to improve identification precision as evidenced by a decreased variance. This was achieved through passive control implemented by changing structural parameters of the story not being currently identified. While this does not represent a feasible full scale control strategy, it does motivate future work on more feasible control strategies like active and semi-active control devices. However, due to the undesirable dynamics and poor performance of the associated AMD, it is recommended to perform future studies on a different structure.

Chapter 9

University of Connecticut Structure

Experimental testing of substructure identification is performed on a four story steel structure in the Advanced Hazards Mitigation Laboratory at the University of Connecticut in partnership with Professor Richard Christenson, Dr. Zhaoshuo Jiang, and Mr. Gannon Stromquist-LeVoir. The preliminary aspects of this work were published in DeVore et al. (2011, 2012). This experiment is designed to validate the methods presented in Chapter 4 and find the limitations within a damage detection context.

This chapter is organized to first describe the experimental apparatus including the structure, shake table, and sensors used in the experiment. Second, an identification error prediction is provided. Third, the experimental procedure is developed where different damage scenarios are described. Fourth, results are presented that compare the effectiveness of substructure identification to other global identification methods. Finally, a brief numerical study is performed to demonstrate the potential identification performance improvement using an active control device.

9.1 Experimental Apparatus

The experimental apparatus is selected to evaluate substructure identification using a shear building estimator. As such, it is important to design the structure to exhibit behavior similar to an idealized shear building. Furthermore, the input excitation and response measurements should be similar to real-world conditions encountered during SHM.

This section will describe the different components of the experimental apparatus and will compare the selected components to real-world situations. The section is organized to first describe the building used as the testbed structure. Next, the shake table used for base excitation is described and the input ground motion is characterized. Finally, the sensors used to measure the response are described and their performance is documented.

9.1.1 Structure

The structure is designed as a uniform four story, single bay moment frame structure that is symmetric in both plan directions. The structure is formed by supporting steel floor plates by four threaded rod columns at the corners along the perimeter. At each floor level, nuts with lock washers are attached on the top and bottom and tightened to enforce clamped connections. This design ensures that the columns independently behave as fixed-fixed columns at each floor. The floor plate stiffness is much higher than the column stiffness, resulting in a structure that behaves like an ideal shear building.

The structure is 3.65 m (12 ft.) tall and the columns are continuous 0.0254 m (1 in.) diameter steel threaded rods. At each floor level, two 0.6096 m \times 0.6096 m \times 0.0127 m (2 ft. \times 2 ft. \times 1/2 in.) solid steel plates are clamped together to form a single floor. The clamping force is provided by a single nut and lock washer installed on the top and



Figure 9.1: University of Connecticut medium-scale structure

k_i	256.5 kN/m
Ci	87.2 N/(m/s)
m_i	88.6 kg
$\omega_{0,i}$	8.56 Hz

Table 9.1: Nominal Story Parameters for Four Story Structure

bottom of each floor and each column. The lock washers ensure that a consistent clamping force is provided and that each column connection behaves rigidly. The columns are attached directly to the shake table and clamped with nuts and lock washers to provide a rigid connection.

The floor levels are uniformly distributed, which gives a nominal story height of 0.9144 m (3 ft.). Moreover, because the columns are rigidly connected at each floor level, the continuous threaded rod can be treated as four independent fixed-fixed columns in each story. This allows easy computation of the equivalent story stiffness k_i and the use of a 4-DOF ROM. Damping is assumed to be proportional to stiffness and lightly damped at approximately 1%. Likewise, the mass of the floor plates (74.1 kg), is much larger than the mass of the columns (14.5 kg) so the mass can be treated as lumped at each floor level. Using nominal dimensions and common values of steel material parameters (Young's modulus 200 GPa and density 7850 kg/m³), the nominal story parameters are calculated and shown in Table 9.1.

9.1.2 Shake Table

The shake table is a 3 ton, uniaxial, $2 \text{ m} \times 2 \text{ m}$, Shore Western shake table with a 20 Hz bandwidth. It is used in displacement control to provide band-limited white noise base excitation to the structure. The command signal is generated by a Simulink model that



Figure 9.2: PSD of Shake Table Acceleration

pre-filters the signal to achieve a nominally flat shake table acceleration response across the structure's bandwidth. In practice this was difficult to achieve as the shake table is commanded in displacement control. The PSD of the shake table acceleration is shown in Figure 9.2.

From Figure 9.2, the magnitude of the shake table acceleration is within 15 dB throughout the bandwidth of the structure. Also, the minimum magnitude is at approximately 5 Hz, which is near the observed interstory natural frequency. Thus, the minimum occurs near the frequency value that is predicted to cause the most error in substructure identification. This is an unavoidable result of the pre-filtering and can be interpreted as providing a conservative bound on the performance of substructure identification.

The power of the excitation can be charactorized by the standard deviation σ_{ii_g} of the shake table acceleration¹ and the peak ground acceleration (PGA). For each of the experiments, the acceleration signal was filtered with a second order Butterworth low-pass

¹This is equivalent to the root mean square of a zero mean signal.



Figure 9.3: Picture of installed accelerometer

filter with cutoff at 20 Hz. The average ground acceleration standard deviation $\sigma_{\ddot{u}_g}$ was 0.0356 m/s² and the average PGA was 0.2270 m/s². This corresponds to a moderate earthquake and is approximately equal to 4.0–5.0 moment magnitude.

9.1.3 Sensors

To measure the response of the structure, accelerometers are installed on each floor including the ground floor. PCB capacitive accelerometers are installed along the plan centerline of each floor in the direction of excitation. The accelerometers installed on the structure have a maximum range of 20 *g* and the accelerometer installed on the shake table has a range of 3 *g*. The acceleration time histories are collected by a Data Physics Data Acquistion system sampled at 256 Hz for 512 seconds. Once the time histories are collected, they are post-processed by subtracting the mean and scaling the signal into engineering units.

To determine the performance of the accelerometers, a noise analysis is performed. A full set of time histories are collected while the structure remains motionless. Using this

Table 9.2: Accelerometer signal to noise ratios.

	üg	\ddot{x}_1	\ddot{x}_2	<i>x</i> ₃	\ddot{x}_4
SNR [dB]	44.14	15.19	26.76	39.29	42.99
NSR [%]	0.62	17.39	4.59	1.09	0.71

record, the SNR can be computed. In this study, the SNR is defined as the RMS response at each floor divided by the RMS of the quiescent time history. In practice, this is computed using the standard deviation, which is equivalent to the RMS of a zero-mean signal. Thus, the SNR is

$$SNR_i = \frac{\sigma_{\ddot{x}_i}}{\sigma_{\ddot{x}_i^{(q)}}}$$
(9.1)

where $\sigma_{\vec{x}_i}$ is the standard deviation of the i^{th} acceleration response and $\sigma_{\vec{x}_i^{(q)}}$ is the standard deviation of the corresponding quiescent response.

The computed SNR of each accelerometer, along with its inverse, the noise to signal ratio (NSR), is shown in Table 9.2. From this table, accelerometers on the ground, third and fourth floors have good performance with a SNR of approximately 40 dB. However, the accelerometers on the first and second floor have much worse performance. It is unclear why these two accelerometers had lower SNR than the other accelerometers though it was not found to substantially effect the results.

9.2 Error Analysis

An error analysis consistent with Section 5.4 using the nominal parameters (Table 9.1) is performed to determine the expected identification error for each floor. The results shown in Figure 9.4 demonstrate that the second story has several orders of magnitude



Figure 9.4: Bar plot of predicted error for four story structure using a logarithmic scale. The particular value does not have physical meaning or units, so one floor's performance should be interpreted in relation to the others.

higher expected error than the other stories. As discussed previously, high expected error indicates that there will be problems with identification at this story level.

To overcome the high expected error in the second story identification, it is necessary to use a control device. In the next two sections, structural control is not implemented; however, it is a source of interest for future work for which preliminary studies are reported in Section 9.5.

9.3 Experimental Procedure

To successfully investigate the performance of substructure identification, an experimental procedure is selected to detect small changes in stiffness within a controlled setting. Care was taken to perform each of the tests identically and only make minor changes that are the subject of the experiment. This section will describe the common practice within each

test, including the excitation, response collection, and signal processing. Also discussed are the different damage scenarios and how they differ from one another.

During each test, the structure is excited by base motion provided by the shake table. The shake table motion is gradually ramped up to a consistent level and then the structure is allowed to vibrate and achieve a stationary response. Because the displacement is small, the structure's response remains linear and elastic throughout the test. Once the response is visually observed to be stationary, the DAQ system is started and a full 512 second time history is collected while the structure vibrates under continued excitation. After the time history is fully collected, the shake table motion is ramped down to zero and the structure's motion is allowed to decay. Only when the structure is completely motionless is another test initiated. This ensures that no spurious motion is incoporated into the collected time histories.

Following a test, the collected data is processed in a consistent, automatic manner. The signals are processed to remove any bias and scaled to express the signals in physical engineering units. Then, the signals are sent to a series of functions that compute the component FRFs consistently. The long time histories are broken into overlapping segments of 67% overlap with a half-range sine window function to create an ensemble of 94 segments. The ensemble is transformed with a 2¹² point DFT and combined to find the component FRFs using Welch's method. Then the FRFs are combined using the substructure estimator to identify the story level stiffness and damping parameters using nonlinear regression. If it is observed that nonlinear regression is unsuccessful, the analyst will re-try the regression, possibly with a different initial estimate. In practice, the identification was usually successfully and the author only re-ran the regression in the

	Tuble 7.5. Dulluge Secturios
\mathcal{D}_0	Undamaged Nominal Structure
\mathcal{D}_1	Two Nuts Loosened on Second Story Column
\mathcal{D}_2	Two Nuts Loosened on Third Story Column
\mathcal{D}_3	Two Nuts Loosened on Fourth Story Column

Table 9.3: Damage Scenarios

initial identification steps before the initial guess and parameter bounds were set. In no case was the data re-processed.

The initial guess is found using the nominal story parameters in Table 9.1. It is observed that these nominal parameters over-predict the stiffness of the structure by over-estimating the stiffness parameter and under-estimating the mass parameter. Therefore, after successful identification of the undamaged case, the stiffness and damping parameters are updated with the identified values. Because the substructure estimator is formulated in terms of mass-normalized values, the mass ratio from one floor to another is the only information about the mass that is needed. Moreover, because the structure is uniform, the mass ratio is identically one and the stiffness and damping parameters can be discussed as mass-normalized.

After computing the stiffness and damping parameters for the undamaged structure, the structure can be modified to simulate damage. The damage pattern selected is the softening of one story by loosening the nuts clamping the floor plate. For instance, for damage case 2, denoted D_2 , the third story stiffness is decreased by loosening the nuts on the top of the second floor. This changes the boundary condition of the base of the third story column from fully restrained to partially restrained. This results in decreased stiffness of the third floor that is observed in the results. For each of the damage cases, exactly two nuts are loosened to maintain symmetry in the direction of motion and



Figure 9.5: Damage detection results showing the mass-normalized identified stiffness value \hat{k}_i/m_i with 95% confidence interval.

to minimize the amount of stiffness loss. The different damage scenarios are listed in Table 9.3.

9.4 Results

The collected data will be analyzed using substructure identification and compared to the global natural frequencies. This comparision will serve to demonstrate the benefits of substructure identification including decentralized computation, damage localization,



Figure 9.6: Substructure identification for undamaged case \mathcal{D}_0 .

and damage sensitivity. This section will present the identification results in a damage detection context.

Using the procedure detailed in Section 9.3, the stiffness and damping parameters of the entire structure are estimated using substructure identification. Figures 9.6, 9.7, 9.8 and 9.9 show the results of the identification for each story at each damage scenario. Additionally, Figure 9.5 shows the identified normalized stiffness value for each story grouped by damage scenario. Moreover, the 95% confidence interval is computed and plotted as an error bar. Each damage scenario represents an independent test and the error bars are predicted using one realization.

The most important identification is the baseline or undamaged case (D_0). This scenario serves as the comparision against which damage can be inferred. In Figure 9.6, it



Figure 9.7: Substructure identification for second story damage case \mathcal{D}_1 .

is clear that the first, third, and fourth stories are successfully identified. However, the second story is improperly identified and H_{EST} does not have the same form as H_{MOD} . This behavior is predicted by the error analysis. This same story will exhibit difficulties in analysis for each damage scenario.

The first damage scenario considered is \mathcal{D}_1 , which corresponds to damage in the second story. As expected, the second story is not successfully identified, so damage cannot be detected. However, this scenario is an excellent test to see if the other stories are effected by damage. Neither the first, third nor fourth stories exhibit much deviation from the undamaged values. This is easy to see in Figure 9.5 where the identified values are close to each other and have small error bars. This result demonstrates that substructure identification is resistant to Type II error (false positives).



Figure 9.8: Substructure identification for third story damage case D_2 .

The second damage scenario D_2 corresponds to damage in the third story. By examining Figure 9.5, it is clear that the third story stiffness is identified as 25% smaller than the undamaged case and it has a small error bar. Therefore, damage is detected in the third story with statistical significance. Moreover, the fourth story stiffness is confidently identified near its undamaged value; so, it is resistant to Type II error. However, the first story shows some difficulties as evidenced by the large error bar and the mis-identified value shown in Figure 9.8. This result shows that something occured to introduce uncertainty into the first story identification. Interestingly, significant levels of torsional vibration were noted for this damage scenario alone. It is hypothesized that the particular global dynamics of this structural configuration couples with one of the torsion modes of the structure. This introduced out-of-plane motion that resulted in an improper identification



Figure 9.9: Substructure identification for fourth story damage case D_3 .

of the first story. The torsion was only noticed visually and was not measured due to limited sensor availability.

The final damage scenario \mathcal{D}_3 considered damage to the fourth story. Consistent with the previous results, damage is detected in the fourth story where the stiffness is decreased by 25%. Likewise, the first and third stories remain undamaged and the second story is improperly identified.

The results of these four scenarios indicate that substructure identification is effective at detecting local damage. Moreover, the identification is sensitive to damage occuring at the story level while remaining insensitive to damage in other stories. This indicates that substructure identification is able to minimize Type II error while still detecting damage. Moreover, Type I error (false positives) is minimized through the use of a linear confidence interval. This interval will be large when a story level is improperly identified and can be used to reject false positives.

These results also indicate two problems with substructure identification of this structure. First, substructure identification is unable to identify the second story parameters. This is the result of low interstory acceleration response, as predicted by the error analysis, and the low levels of damping in the structure ($\zeta < 0.01$). This result should not serve to exclude substructure identification but rather motivate the use of controlled substructure identification to overcome this deficiency by using a control device to temporarily change the structure's response. The second problem raised by the results is improper identification of the first story in the second damage case (D_2). As stated previously, this is likely a result of the coupled torsion response; additional sensors could be used to remove the torsional component of the response. Likewise, the first floor accelerometer suffers from the highest levels of noise, so it is susceptible to spurious response.

The results of substructure identification can be compared to global identification methods. The most simple frequency-domain global identification method tracks the global natural frequencies. Using the acceleration time histories, the FRFs $H_{\ddot{x}_i \ddot{u}_g}$ of input ground acceleration to output response are computed. Then, using the fourth floor FRF $H_{\ddot{x}_i \dot{u}_g}$, the natural frequencies are identified by picking the peak magnitude response,

	ω_1	ω_2	ω_3	ω_4
\mathcal{D}_0	2.06	6.25	10.25	12.81
\mathcal{D}_1	1.75	6.00	10.12	12.62
\mathcal{D}_2	1.88	6.12	9.62	12.81
\mathcal{D}_3	2.00	5.50	10.06	12.75

Table 9.4: Global natural frequencies of the structure by damage scenario in Hz.

	k_1/m_1	k_2 / m_2	k_3 / m_3	k_4 / m_4
π	1637	NI / A	1394	1604
ν_0	(1630,1643)	1N/A	(1388,1400)	(1596,1612)
Д.	1611	NI / A	1417	1658
ν_1	(1603,1619)	1N/A	(1414,1420)	(1651,1664)
\mathcal{D}_{a}	1521	NI / A	1073	1681
ν_2	(1473,1570)	1 N / A	(1069,1078)	(1672,1689)
\mathcal{D}_3	1635	NI / A	1538	1234
	(1631,1640)	11/11	(1527,1549)	(1224,1244)

Table 9.5: Identified substructure mass-normalized stiffness parameters (k_i/m_i) with 95% confidence intervals. Damaged stories are shown in bold.

recorded in Table 9.4, for each damage scenario. Damage detection is performed by noting a decrease in one or more of the natural frequencies. Damage is detected in scenario D_1 by a decrease of 15% in the first natural frequency. For the second damage scenario, a decrease of 9% is noted in the first natural frequency. Finally, in the third damage scenario, damage is detected by a decrease of 12% in the second natural frequency.

By comparing the damage sensitivities, the the performance of the two damage detection methods can be compared. With the exception of \mathcal{D}_1 , substructure identification is over twice as sensitive to damage as the global natural frequency. Additionally, substructure identification provides damage localization whereas global natural frequencies cannot provide damage localization without additional modal information. In the first damage scenario \mathcal{D}_1 , global natural frequencies out-perform substructure identification because the substructure identification method is incapable of identifying the second story parameters.

Table 9.6: Design specifications of various controllers as specified by various constraints. Note: the force constraint is specified as a percentage of the total weight of the structure.

	σ^{c}	ζ ^c	f^{c}	$h_2^{\rm c}$	h_{∞}^{c}
Name	[rad/s]	[%]	[%]	[dB]	[dB]
CT00	-5.0	10	2	40	40
CT01	-5.0	10	2	20	40
CT02	-5.0	10	2	40	20
CT03	-5.0	5	2	40	40

9.5 Preliminary Study of Active Control

Using the results of the previous sections, a preliminary study of controlled substructure identification can be performed on a ROM of the structure. The second story was not experimentally identified by substructure identification so the controller will be designed to improve substructure identification of the second story. A suite of controllers will be designed and tested with different sensor configurations using the procedure detailed in Chapter 7. Then, the performance of controlled substructure identification in undamaged and damaged configurations will be determined using MCS.

This study will design a controller for an AMD installed on the top floor. It is assumed that the secondary controller of the AMD has high-authority and that the control system has a static gain. This assumption is unlikely to be true in practice, but further considerations are outside the scope of this study.

The controller will be designed by following the control design procedure in Section 7.3.2 using only the controllers with the most restrictive constraints. These controllers are specified in Table 9.6. This selection is purposeful because the low damping encountered in the structure will likely decrease the robustness of high gain controllers. By restricting the design space to restrictive constraints, high gain controllers are avoided.



Figure 9.10: Second story interstory acceleration response for various controllers defined in Table 9.6. The center of the identification bandwidth is shown as a dashed line.

After designing the controllers, the system performance of the closed loop system is shown in Table 9.7. Likewise, the second story interstory acceleration response is shown in Figure 9.10. The figure shows a modest increase in the second story interstory acceleration response, which is reflected in the first column of Table 9.7. An interesting side-effect of the controller is that the overall system response is decreased and the stability is increased when compared to the uncontrolled system.

To determine the substructure identification performance of the controllers, MCS is used. 10,000 independent experiments of the undamaged and damaged scenarios. The damage scenario is selected to be D_1 with 25% stiffness loss applied to the second story. The second story stiffness is identified and the statistics are shown in Table 9.8.

	J	$\max_i \sigma_i$	$\min_i \zeta_i$	$ u_c _2$	$ T^{c} _{2}$	$ T^{c} _{\infty}$	$\max_i \ \delta_i\ _2$	$\max_i \ \ddot{x}_i\ _2$
Name	[·]	[rad/s]	[%]	[%]	[dB]	[dB]	[mm]	$[m/sec^2]$
UNC	0.92	-0.06	0.34	0.00	37.19	48.78	5.23	4.61
CT00	1.10	-5.00	9.95	2.00	23.57	13.99	0.67	0.84
CT01	1.02	-4.92	9.84	3.82	24.19	11.97	0.83	1.14
CT02	1.10	-5.00	9.95	2.00	23.57	13.99	0.67	0.84
CT03	1.12	-5.00	6.16	2.00	23.93	14.01	0.65	0.89

Table 9.7: Design results of various controllers compared to the uncontrolled case. Note: the force constraint is specified as a percentage of the total weight of the structure.

Table 9.8: Statistics of substructure identification of the second story stiffness presented as percentage of the nominal stiffness value. The damaged case corresponds to a decrease in stiffness of 5%.

	Undan	naged	Dama	ged
	Mean STD		Mean	STD
UNC	-0.69	12.08	-26.04	5.83
CT00	-0.89	7.61	-25.16	2.11
CT01	-0.72	7.18	_	-
CT02	-0.85	7.37	-25.18	2.34
CT03	-0.97	7.27	-25.30	3.08

The results show three findings. First, controlled substructure identification is effective at decreasing the error variance of second story stiffnees identification. The improvement is approximately a two-fold decrease. Second, the bias is slightly increased for the undamaged scenario. Third, CT01 results in an unstable system for the damaged scenario indicating the controller's lack of robustness.

CT00 is selected as the controller and its performance is tested in a variety of different sensor configurations. These configurations and the system performance is shown in Table 9.9. The second story interstory acceleration response is shown in Figure 9.11. The results show that the different configurations have similar amplification of the second story interstory acceleration response.

Table 9.9: Design results of various observers. The first columns refer to which story-level acceleration sensors are used with zero corresponding to the ground acceleration.

	Obs. Vec.	J	$\max_i \sigma_i$	$\min_i \zeta_i$	$ u_c _2$	$ H _{2}$	$ H _{\infty}$
Name		[·]	[rad/s]	[%]	[%]	[dB]	[dB]
UNC	-	0.92	-0.06	0.34	0.00	37.19	48.78
CT00	full state	1.10	-5.00	9.95	2.00	23.57	13.99
OB00	full accel.	1.10	-5.00	5.60	2.00	23.56	13.99
OB01	[0,3,4]	1.10	-3.68	3.60	2.00	23.56	13.99
OB02	[0,4]	1.10	-2.17	2.14	2.00	23.56	13.99
OB03	[3,4]	1.00	-5.00	9.95	1.77	23.22	14.07
OB04	[4]	0.93	-5.00	8.80	1.70	23.62	14.32



Figure 9.11: Second Story Interstory Acceleration Response for various observers.

0	0							
	Undan	naged	Dama	ged				
	Mean	STD	Mean	STD				
UNC	-0.69	12.08	-26.04	5.83				
CT00	-0.89	7.61	-25.16	2.11				
OB00	-0.70	7.04	-25.18	2.33				
OB01	-0.78	7.32	-25.14	2.12				
OB02	-1.06	7.95	-25.18	2.37				
OB03	-1.43	8.91	-24.96	2.06				
OB04	-1.13	8.27	-25.43	3.71				

Table 9.10: Statistics of controlled identification with observers of the second story parameters presented as percentages.

Once the observers are designed, substructure identification performance is determined for the undamaged and damaged scenarios. The results are summarized in Table 9.10. The results present two findings. First, sensor configurations including the ground acceleration are able to provide identification performance comparable to the full state controller. This will likely result in successful identification of the second story stiffness. Second, sensor configurations excluding the ground acceleration have worse identification performance and see an increase in bias and error variance. The author is uncertain as to whether the second story stiffness will be successfully identified.

9.6 Findings

Substructure identification is performed on a four story steel structure. Damage is successfully detected with statistical significance for most scenarios. Moreover, damage is successfully localized to the damage location in each successful identification. This serves to confirm substructure identification for damage detection in a shear building. Furthermore, substructure identification is found to be more sensitive to damage than global modal measures. Error analysis predicts that the second story will not be properly identified. This behavior is indeed observed: in no cases is the second story successfully identified. This result motivates future work to use a structural control device to improve identification accuracy.

Chapter 10

Conclusions

This study has developed a generalized procedure for substructure identification, including tools to analyze and predict a particular estimator's performance. This was accomplished within the context of damage detection with appropriate examples and simulations performed to demonstrate performance. Experimental verification was performed and found to confirm theoretical and simulated results. This was accomplished in three parts.

First, a generalized procedure was developed with guidelines for forming a ROM and substructure estimator. Practical concerns surrounding FRF estimation were described and best practice outlined. An extensive study of nonlinear regression techniques including confidence intervals/regions was presented. It was found that a LSE combined with a Jacobian based linear confidence interval provides the best statistical performance. Following, the statistical curvature was derived and computed for a shear structure estimator to demonstrate its satisfactory performance. Finally, an error analysis was derived for the LSE; this error analysis allows for control design and optimal sensor placement. Next, numerical simulation was used to demonstrate the performance of the substructure estimator. First, uncertainty propagation was described through Monte Carlo simulation. Continuing, Monte Carlo simulation was used to detect small changes in stiffness and was shown to properly locate damage with good Type I/II error performance.

Then, a control design procedure was used to develop a controller for an AMD to temporarily improve the substructure identification performance of a particular substructure. Various controllers designed using different constraints were compared and their substructure identification performance was characterized through MCS. Then, a controller was tested using various realistic sensor configurations. Controlled substructure identification was found to have a dramatic positive effect on substructure identification performance.

Finally, experimental verification was provided through two studies. The first study utilized a bench-scale, two-story structure subject to white noise base excitation. This study found that passive control methods can improve identification accuracy by altering the global dynamics of the structure. The second study clearly demonstrated the damage detection potential of substructure identification. It was possible to detect a small amount of damage simulated by releasing the boundary conditions of one story level of a 12 ft, four-story, steel structure. This damage was not only detected but localized to the correct story level in all but one case: the second-floor identification, which was predicted by the identification error analysis to fail. This positive confirmation of the identification error analysis motivates future controlled substructure identification described in Section 9.5.

Chapter 11

Future Work

Future work is required to extend the developed controlled substructure identification techniques to physical systems. The next step is to perform active control experiments using the University of Connecticut structure. Realistic controllers were developed and their performance described in Section 9.5. Section 11.1 will desribe the necessary next steps.

Controlled substructure identification can be further extended to semi-active control devices. To do this, an alternative design procedure will be needed that can parameterize and optimize the inherently nonlinear system. Section 11.2 will describe these steps further.

This study focused entirely on ROMs that admitted only translational DOFs. This results in ROMs that behave as a shear beam/building. Future work will be needed to investigate ROMs admitting rotational DOFs to investigate other structures that have more complicated deformations. For example, a moment frame structure can be analyzed such as a welded steel moment frame (WSMF) and bridge structures. Sections 11.3 and 11.4 will describe future research avenues.
11.1 University of Connecticut Active Control Experiment

A second round of testing will be performed at the University of Connecticut to demonstrate the effectiveness of controlled substructure identification. This experiment will use the control design procedure described in Chapter 7. However, there will be several unique issues that need to be overcome.

First, the low levels of damping ($\zeta_{0,i} \approx 1\%$) makes substructure identification difficult because the response is highly peaked resulting in fewer frequency points being utilized. This reduces the effective number of statistical degrees of freedom, which introduces greater error variance. Additionally, the low damping makes any controller susceptible to structural parameter uncertainty and decreases the overall robustness of the controller. This means that controllers performing well for the undamaged case can become unstable with small changes in stiffness (damage). The robustness of the proposed controller will need to be studied explicitly.

Second, any control device will admit control system dynamics that can affect the closed loop performance. Substructure identification requires higher frequency bandwidth, which may be attenuated by some control system dynamics. Physical implementations will need to determine if control system dynamics will effect the closed loop performance and, if so, find ways to design around these limitations. A potential path forward is to include control system dynamics in the computation of the objective function so that the control gain matrix is designed incorporating the control system dynamics.

Third, control structure interaction (CSI) can decrease the authority of a controller if feedback dynamics between the control device and structure are not accounted for (Dyke et al., 1995). This will be of special concern for substructure identification controllers because they are specifically designed to apply control forces at the poles and zeros of a structure. Therefore, in addition to designing for the control system dynamics, special care will need to diagnose and overcome any CSI that is encountered.

Section 9.5 describes the performance of several possible controllers to be applied to the University of Connecticut structure. The selected controller is then tested using a variety of sensor configurations and shown to improve substructure identification. A successful active control experiment will utilize this study as the starting point to test for the described issues above. After diagnosing and compensating for any issues, the controller should be implemented in the experimental testbed to determine if second story damage can be detected. Successful damage detection will prove the viability and effectiveness of controlled substructure identification in this configuration.

11.2 Semi-Active Controlled Substructure Identification

Semi-active control devices such as magnetorheological (MR) dampers are commonly used in civil structural control applications. These devices offer greater adaptability than similar passive devices while remaining inherently stable and using less energy than active devices. Because of the widespread utility of semi-active devices, controlled substructure identification should be investigated using semi-active control devices.

Control design using a semi-active device requires modifications from the active control design presented in Chapter 7. First, the closed loop system is nonlinear because of the semi-active control device. This means that the objective functional can not be computed directly. Modifications will be needed to determine an alternative. Second, the closed loop poles of the system cannot be used as the search space. The control gain matrix will need to be used directly as the search space. As a result of these complications and others, it is likely that semi-active control design for substructure identification will be highly non-convex.

Semi-active control does offer a unique benefit to controlled substructure identification. The closed loop system of a semi-active controller is inherently stable because the control device can only dissipate energy. Therefore, active controllers that would otherwise result in an unstable system can be used in a semi-active system that will remain stable.

Future work will be necessary to find ways of computing the objective functional for various controllers. Finding the optimal controller will require non-convex optimization that may be computationally expensive. Future work will need to find ways to efficiently cover the search space.

11.3 Damage Detection in a Welded Steel Moment Frame

Using a WSMF as the testbed structure, damage will be simulated by the presence of a plastic hinge. This corresponds to observed damage behavior in WSMFs constructed before the 1994 Northridge earthquake. This will be accomplished through the use of a nonlinear finite element model (FEM). The specifics are described herein.

Prior to the Northridge earthquake, WSMFs were constructed with moment connection details that contained a welded backing bar which was left in place. Research following the earthquake (Kim et al., 2003) demonstrates that these connections are at risk of brittle fracture in the flange welds. This results in a 22–44% decrease of plastic moment capacity.

The post-fracture behavior of the joint will be simulated by a nonlinear FEM. This will be used to generate acceleration time-histories for damage detection. These time histories will be analyzed in the same way as a shear building using a similar estimator to the one developed in Chapter 4. The frame structure estimator will need to incorporate rotational DOFs which requires additional terms.

It is necessary to develop a ROM; for this example, static condensation is used. The full-order FEM is reduced to a 3DOF chain structure. This is accomplished by lumping the story mass at one point and reducing the stiffness matrix using static condensation. Then, the substructure estimator is developed and its damage detection performance can be found using MCS.

11.4 Continuous Beam Estimator for Bridge Structures

The final major research thrust is to develop an estimator appropriate for bridges. This represents an area of substructure identification research that has not been explored. To successfully develop a continuous beam estimator, two obstacles need to be overcome.

First, Bernoulli beams require both displacements and rotations. Displacements can be accommodated with measured accelerations, however, rotations cannot be easily measured. Therefore, rotations will need to be estimated via a dense set of acceleration measurements. Alternatively, the beam can be instrumented such that the acceleration sensors are placed at rotation nodes so that they will have minimal response in the identification frequency bandwidth.

The second obstacle is interface forces. A successful estimator requires that the equation of motion for the element consider be balanced. This requires that the interface force be measured, which requires prior identification that is difficult to initiate in an unmeasured structure. However, this may be overcome by using two overlapping elements with a common interface. The interface forces will be estimated using each element and then set equal to each other. The optimization variable is the stiffness, *EI*, which can be found by minimizing the difference between the interface forces. It will be important to develop a technique to locate the most effective amount of overlap between the two elements. This will be accomplished using the expected identification error for continuous structures following a similar procedure to the one in Section 4.4.

Once an appropriate beam estimator is developed, numerical simulation will be used to demonstrate its validity. Special care will be taken to detect and localize damage. Furthermore, simulation will be used to demonstrate an optimal sensor placement strategy as found through the minimization of the expected identification error. This will be taken as the analogue to controlled identification in a shear building.

Finally, using *in situ* bridge measurements, substructure identification will be performed on a physical bridge. The bridge to be identified is currently monitored through a joint research project between the University of Connecticut and the Connecticut Department of Transportation. Using data collected from the bridge, the stiffness will be identified by using a ROM, which will be computed with the accelerometer sensor location and plan drawings. Several independent identifications will be performed over several days to compute the identification variance, which will be statistically controlled via measured environmental conditions. This will demonstrate the viability of substructure identification for long-term bridge monitoring.

Bibliography

- B M Adams, K R Dalbey, M S Eldred, D M Gay, L P Swiler, W J Bohnhoff, J P Eddy, K Haskell, and P D Hough. DAKOTA: A Multilevel Parallel Object-Oriented Framework for Design Optimization, Parameter Estimation, Uncertainty Quantification, and Sensitivity Analysis Version 5.0 User's Manual. Technical Report, Sandia National Laboratories, Albuquerque, NM, 2010.
- J Antoni and J Schoukens. A Comprehensive Study of the Bias and Variance of Frequency Response Function Measurements: Optimal Window Selection and Overlapping Strategies. *Automatica*, 43(10):1723–1736, October 2007. doi: 10.1016/j.automatica.2007.02.020.
- ASCE. Infrastructure Report Card. Technical Report, American Society of Civil Engineers, January 2009.
- ATC. ATC-20: Procedures for Postearthquake Safety Evaluation of Buildings. Technical Report, Applied Technology Council, Redwood City, CA, 1989.
- D M Bates and D G Watts. Relative Curvature Measures of Nonlinearity. *Journal of the Royal Statistical Society: Series B (Methodological)*, 42(1):1–25, 1980.
- D M. Bates and D G. Watts. *Nonlinear Regression Analysis and Its Applications*. John Wiley & Sons, New York, May 2007. doi: 10.2307/1268866.
- E M L Beale. Confidence Regions in Non-Linear Estimation. *Journal of the Royal Statistical Society: Series B (Methodological)*, 22(1):41–88, 1960.
- Bridges, Structures, and Hydraulics, 23, C.F.R. pt. 650 (2011).
- J M W Brownjohn. Structural Health Monitoring of Civil Infrastructure. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 365 (1851):589–622, February 2007. doi: 10.1098/rsta.2006.1925.
- R A Bruce and S K Tubbesing. The Loma Prieta, California, Earthquake of October 17, 1989–Loss Estimation and Procedures. Technical Report, United States Geological Survey, Washington DC, 1994.
- J B Burl. *Linear Optimal Control:* H_2 and H_{∞} Methods. Addison Wesley Longman, Inc, Menlo Park, CA, 1st Edition, 1999. ISBN 0-201-80868-4.
- J M Caicedo, S J Dyke, and E A Johnson. Natural Excitation Technique and Eigensystem Realization Algorithm For Phase I of the IASC-ASCE Benchmark Problem: Simulated Data. *Journal of Engineering Mechanics*, 130(1):49–60, 2004.

- A K Chopra. *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. Prentice Hall, Upper Saddle River, NJ, 2nd Edition, 2001.
- R R Craig and M C C Bampton. Coupling of Substructures for Dynamic Analyses. *AIAA Journal*, 6(7):1313–1319, 1968.
- C DeVore and E A Johnson. Damage Detection and Uncertainty Quantification for Substructure Identification. *in preparation*, 2013.
- C DeVore, E A Johnson, and R E Christenson. Experimental Verification of Controlled Substructure Identification using Control Devices : A Preliminary Simulation Study. In *Proceedings of 8th International Workshop on Structural Health Monitoring*, Stanford, CA, 2011.
- C DeVore, Z Jiang, E A Johnson, R E Christenson, and G Stromquist-Levoir. Experimental Verification of Substructure Identification for Damage Detection. In *Proceedings of the* 2012 Joint Conference of the Engineering Mechanics Institute and the 11th ASCE Joint Specialty Conference on Probabilistic Mechanics and Structural Reliability, South Bend, IN, 2012.
- S W Doebling, C R Farrar, M B Prime, and D W Shevitz. Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in Their Vibration Characteristics: A Literature Review. Technical Report, Los Alamos National Laboratory, 1996.
- J R Donaldson and R B Schnabel. Computational Experience With Confidence Intervals and Confidence for Nonlinear Least Squares. *Technometrics*, 29(1):67–82, 1987.
- S J Dyke, B F Spencer Jr., P Quast, and M K Sain. Role of Control-Structure Interaction in Protective System Design. *Journal of Engineering Mechanics*, 121(2):322–338, February 1995. doi: 10.1061/(ASCE)0733-9399(1995)121:2(322).
- C R Farrar and K Worden. An Introduction to Structural Health Monitoring. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences,* 365 (1851):303–15, February 2007. doi: 10.1098/rsta.2006.1928.
- W E Gates and M Morden. Lessons from Inspection, Evaluation, Repair and Construction of Welded Steel Moment Frames following the Northridge Earthquake, 1995.
- M R Hernandez-Garcia, S F Masri, R Ghanem, E Figueiredo, and C R Farrar. An Experimental Investigation of Change Detection in Uncertain Chain-like Systems. *Journal of Sound and Vibration*, 329(12):2395–2409, June 2010. doi: 10.1016/j.jsv.2009.12.024.
- J Hou, Ł Jankowski, and J Ou. A Substructure Isolation Method for Local Structural Health Monitoring. *Structural Control and Health Monitoring*, 18(6):601–618, October 2011. doi: 10.1002/stc.389.
- W C Hurty. Dynamic Analysis of Structural Systems Using Component Modes. *AIAA journal*, 3(4):678–685, 1965.
- J N Juang and R S Pappa. An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction. *Journal of Guidance*, 8(5):620–627, 1985.

- S J Julier and J K Uhlmann. Unscented Filtering and Nonlinear Estimation. *Proceedings of the IEEE*, 92(3):401–422, March 2004. doi: 10.1109/JPROC.2003.823141.
- S B Kim, B F Spencer Jr, and C B Yun. Frequency Domain Identification of Multi-Input, Multi-Output Systems Considering Physical Relationships between Measured Variables. *Journal of Engineering Mechanics*, 131(5):461–472, 2005.
- T Kim, B Stojadinovic, and A S Whittaker. Physical and Numerical Performance Evaluation of Steel Moment Resisting Frames. Technical Report, Pacific Earthquake Engineering Research Center, Berkeley, CA, 2003.
- C G Koh, B Hong, and C Y Liaw. Substructural and Progressive Structural Identification Methods. *Engineering Structures*, 25(12):1551–1563, October 2003. doi: 10.1016/ S0141-0296(03)00122-6.
- C G Koh and K Shankar. Substructural Identification Method Without Interface Measurement. *Journal of Engineering Mechanics*, 129(7):769–776, 2003.
- C G Koh, L M See, and T Balendra. Estimation of Structural Parameters in Time Domain: A Substructure Approach. *Earthquake Engineering & Structural Dynamics*, 20(8):787–801, 1991. doi: 10.1002/eqe.4290200806.
- A Leon-Garcia. *Probability, Statistics, and Random Processes for Electrical Engineering*. Pearson Prentice Hall, Upper Saddle River, NJ, 3rd Edition, 2007. ISBN 978-0-13-147122-1.
- J S Lew. Using Transfer Function Parameter Changes for Damage Detection of Structures. *AIAA Journal*, 33(11):2189–2193, November 1995. doi: 10.2514/3.12965.
- H W Lilliefors. On the Kolmogorov-Smirnov Test for Normality with Mean and Variance Unknown. *Journal of the American Statistical Association*, 62(318):399–402, June 1967.
- J P Lynch and K J Loh. A Summary Review of Wireless Sensors and Sensor Networks for Structural Health Monitoring. *The Shock and Vibration Digest*, 38(2):91–128, March 2006. doi: 10.1177/0583102406061499.
- S Marsili-Libelli, S Guerrizio, and N Checchi. Confidence Regions of Estimated Parameters for Ecological Systems. *Ecological Modelling*, 165(2-3):127–146, July 2003. doi: 10.1016/S0304-3800(03)00068-1.
- J M Mendel. *Lessons in Estimation Theory for Signal Processing, Communications, and Control.* Prentice-Hall, Upper Saddle River, NJ, first edition, 1995. ISBN 0-13-120981-7.
- R V D Merwe. *Sigma-point Kalman filters for Probabilistic Inference in Dynamic State-space Models.* Ph.D. Dissertation, Oregon Health and Science University, 2004.
- J M Nichols, M Seaver, and S T Trickey. A Method for Detecting Damage-induced Nonlinearities in Structures Using Information Theory. *Journal of Sound and Vibration*, 297(1-2):1–16, October 2006a. doi: 10.1016/j.jsv.2006.01.025.

- J M Nichols, M Seaver, S T Trickey, L W Salvino, and D L Pecora. Detecting Impact Damage in Experimental Composite Structures: An Information-theoretic Approach. *Smart Materials and Structures*, 15(2):424–434, April 2006b. doi: 10.1088/0964-1726/15/2/023.
- NTSB. Collapse of I-35W Highway Bridge Minneapolis, Minnesota August 1, 2007. Technical Report, National Transportation Safety Board, 2008.
- L A Overbey and M D Todd. Dynamic System Change Detection using a Modification of the Transfer Entropy. *Journal of Sound and Vibration*, 322(1-2):438–453, April 2009. doi: 10.1016/j.jsv.2008.11.025.
- R Pintelon and J Schoukens. *System Identification: A Frequency Domain Approach*. IEEE, New York, 2001.
- J A Rice. Characterization of Wireless Smart Sensor Performance. Technical Report, University of Illinois at Urbana-Champaign, February 2010.
- S H Sim, B F Spencer Jr, M Zhang, and H Xie. Automated Decentralized Modal Analysis Using Smart Sensors. *Structural Control and Health Monitoring*, 17(8):872–894, 2010.
- A W Smyth, S F Masri, A G Chassiakos, and T K Caughey. On-Line Parametric Identification of MDOF Nonlinear Hysteretic Systems. *Journal of Engineering Mechanics*, 125(2): 133, 1999. doi: 10.1061/(ASCE)0733-9399(1999)125:2(133).
- H Sohn, C R. Farrar, F Hemez, and J Czarnecki. A Review of Structural Health Monitoring Literature : 1996-2001. Technical Report LA-13976-MS, Los Alamos National Laboratory, 2004.
- K F Tee, C G Koh, and S T Quek. Substructural First- and Second-order Model Identification for Structural Damage Assessment. *Earthquake Engineering & Structural Dynamics*, 34(15):1755–1775, December 2005. doi: 10.1002/eqe.500.
- T N Trinh and C G Koh. An Improved Substructural Identification Strategy for Large Structural Systems. *Structural Control and Health Monitoring*, preprint, 2011.
- P Welch. The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms. *IEEE Transactions on Audio and Electroacoustics*, 15(2):70–73, June 1967. doi: 10.1109/TAU.1967.1161901.
- Y Wen. Method for Random Vibration of Hysteretics Systems. *Journal of the Engineering Mechanics Division*, 102(2):249–263, 1976.
- L Xie and A Mita. Using Component Mode Synthesis to Estimate the Restoring Force of an Isolation Layer Subjected to Earthquakes. *Structural Control and Health Monitoring*, 17 (2):152–177, 2010.
- Z Xing and A Mita. A Substructure Approach to Local Damage Detection of Shear Structure. *Structural Control and Health Monitoring*, 19(2):309–318, March 2012. doi: 10. 1002/stc.439.

- J N Yang and S Lin. Identification of Parametric Variations of Structures Based on Least Squares Estimation and Adaptive Tracking Technique. *Journal of Engineering Mechanics*, 131(3):290, 2005. doi: 10.1061/(ASCE)0733-9399(2005)131:3(290).
- G L Yeo and C A Cornell. Post-quake Decision Analysis using Dynamic Programming. *Earthquake Engineering & Structural Dynamics*, 38(1):79–93, January 2009. doi: 10.1002/eqe.842.
- K V Yuen and L S Katafygiotis. Substructure Identification and Health Monitoring using Noisy Response Measurements Only. *Computer-Aided Civil and Infrastructure Engineering*, 21(4):280–291, 2006.
- D Y Zhang and E A Johnson. Substructure Identification for Shear Structures I: Substructure Identification Method. *Structural Control and Health Monitoring*, in press, 2012a. doi: 10.1002/stc.1497.
- D Y Zhang and E A Johnson. Substructure Identification for Shear Structures II: Controlled Substructure Identification. *Structural Control and Health Monitoring*, in press, 2012b. doi: 10.1002/stc.1498.
- D Y Zhang and E A Johnson. Substructure Identification for Shear Structures: Cross Power Spectral Density Method. *Smart Materials and Structures*, 21, 2012c. doi: 10.1088/ 0964-1726/21/5/055006.
- D Y Zhang, C DeVore, and E A Johnson. Experimental Verification of the Use of Control Systems to Improve SHM of Shear Structures. In *Proceedings of 7th International Workshop on Structural Health Monitoring*, Stanford, CA, January 2009.
- D Zhang, C DeVore, and E A Johnson. Experimental Verification of Controlled Substructure Identification for Shear Structures. In *Proceedings of 5th World Conference on Structural Control and Monitoring*, Tokyo, Japan, 2010.
- Q Zhao, T Sawada, and K Hirao. Localized Identification of MDOF Structures in the Frequency Domain. *Earthquake Engineering & Structural Dynamics*, 24:325–338, 1995.